thm_2Erich_list_2EBUTLASTN__CONS (TMJ6NwPbjzYvqgYxcP99J1gXd5n5DucCi5w)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27a})$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{5}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$
$$\tag{6}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$\tag{7}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist$$
$$A\_27a)$$
$$\tag{8}$$

Let $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ESNOC\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$
$$\tag{9}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{10}$$

Definition 14 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Let $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$\tag{11}$$

Let $c\_2Elist\_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EDROP\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum})$$
$$\tag{12}$$

Definition 15 *We define* c_2Erich__list_2EBUTLASTN *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1xs$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))\ (ap\\ \mathsf{c\_2Enum\_2ESUC}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\\V1m))))) \tag{13}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\(ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))\ \mathsf{c\_2Enum\_2E0})))) \tag{14}$$

Assume the following.
$$True \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\\V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in\\A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow\\True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((\\(p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow\\True)) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in\\A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow\\(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(\\p\ V0t)))))))) \tag{22}$$

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Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (((ap \ (c\_2Elist\_2ELENGTH \ A\_27a) \\ (c\_2Elist\_2ENIL \ A\_27a)) = \mathsf{c\_2Enum\_2E0}) \wedge (\forall V0h \in A\_27a.(\\ \forall V1t \in (ty\_2Elist\_2Elist \ A\_27a).((ap \ (c\_2Elist\_2ELENGTH \\ A\_27a) \ (ap \ (ap \ (c\_2Elist\_2ECONS \ A\_27a) \ V0h) \ V1t)) = (ap \ \mathsf{c\_2Enum\_2ESUC} \\ (ap \ (c\_2Elist\_2ELENGTH \ A\_27a) \ V1t))))))$$

(23)

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow ((\forall V0x \in A\_27a.((ap \ (ap \ (c\_2Elist\_2ESNOC \\ A\_27a) \ V0x) \ (c\_2Elist\_2ENIL \ A\_27a)) = (ap \ (ap \ (c\_2Elist\_2ECONS \\ A\_27a) \ V0x) \ (c\_2Elist\_2ENIL \ A\_27a)))) \wedge (\forall V1x \in A\_27a.(\forall V2x\_27 \in \\ A\_27a.(\forall V3l \in (ty\_2Elist\_2Elist \ A\_27a).((ap \ (ap \ (c\_2Elist\_2ESNOC \\ A\_27a) \ V1x) \ (ap \ (ap \ (c\_2Elist\_2ECONS \ A\_27a) \ V2x\_27) \ V3l)) = (ap \ (\\ ap \ (c\_2Elist\_2ECONS \ A\_27a) \ V2x\_27) \ (ap \ (ap \ (c\_2Elist\_2ESNOC \ A\_27a) \\ V1x) \ V3l)))))))$$

(24)

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1l \in \\ (ty\_2Elist\_2Elist \ A\_27a).((ap \ (c\_2Elist\_2ELENGTH \ A\_27a) \ (ap \\ (ap \ (c\_2Elist\_2ESNOC \ A\_27a) \ V0x) \ V1l)) = (ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (\\ c\_2Elist\_2ELENGTH \ A\_27a) \ V1l)))))$$

(25)

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist \ A\_27a)}). \\ (((p \ (ap \ V0P \ (c\_2Elist\_2ENIL \ A\_27a))) \wedge (\forall V1l \in (ty\_2Elist\_2Elist \\ A\_27a).((p \ (ap \ V0P \ V1l)) \Rightarrow (\forall V2x \in A\_27a.(p \ (ap \ V0P \ (ap \ (ap \ (\\ c\_2Elist\_2ESNOC \ A\_27a) \ V2x) \ V1l))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ A\_27a).(p \ (ap \ V0P \ V3l)))))$$

(26)

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p \ (ap \ V0P \ \mathsf{c\_2Enum\_2E0})) \wedge \\ (\forall V1n \in ty\_2Enum\_2Enum.((p \ (ap \ V0P \ V1n)) \Rightarrow (p \ (ap \ V0P \ (ap \ \mathsf{c\_2Enum\_2ESUC} \\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p \ (ap \ V0P \ V2n)))))$$

(27)

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow (\\ (\forall V0l \in (ty\_2Elist\_2Elist \ A\_27a).((ap \ (ap \ (\mathsf{c\_2Erich\_list\_2EBUTLASTN} \\ A\_27a) \ \mathsf{c\_2Enum\_2E0}) \ V0l) = V0l)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. \\ (\forall V2x \in A\_27b.(\forall V3l \in (ty\_2Elist\_2Elist \ A\_27b).(\\ (ap \ (ap \ (\mathsf{c\_2Erich\_list\_2EBUTLASTN} \ A\_27b) \ (ap \ \mathsf{c\_2Enum\_2ESUC} \ V1n)) \\ (ap \ (ap \ (c\_2Elist\_2ESNOC \ A\_27b) \ V2x) \ V3l)) = (ap \ (ap \ (\mathsf{c\_2Erich\_list\_2EBUTLASTN} \\ A\_27b) \ V1n) \ V3l))))))$$

(28)

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Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.($$
$$\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))) \Rightarrow (\forall V2x \in A\_27a.$$
$$((ap\ (ap\ (\mathsf{c\_2Erich\_list\_2EBUTLASTN}\ A\_27a)\ V0n)\ (ap\ (ap\ (c\_2Elist\_2ECONS}$$
$$A\_27a)\ V2x)\ V1l)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x)\ (ap\ (ap\ ($$
$$\mathsf{c\_2Erich\_list\_2EBUTLASTN}\ A\_27a)\ V0n)\ V1l)))))))$$