

thm_2Erich__list_2EBUTLASTN__LASTN
(TMc7WkpD4h9jGTefb1S1WHUUWmkmcHYxpHd)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_Emin_2E_40$

Definition 11 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (5)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (6)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (7)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (8)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (9)$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EREVERSE\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \quad (10)$$

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ETAKE\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 14 We define $c_2Erich_list_2ELASTN$ to be $\lambda A_27a : \iota. \lambda V0n \in ty_2Enum_2Enum. \lambda V1xs \in (ty$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ESNOC\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (12)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Definition 15 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Elist_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EDROP\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 16 We define $c_2Erich_list_2EBUTLASTN$ to be $\lambda A_27a : \iota. \lambda V0n \in ty_2Enum_2Enum. \lambda V1xs$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ c_2Enum_2ESUC\ V0n))\ (ap\ c_2Enum_2ESUC\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0n)\ V1m)))) \quad (15)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ c_2Enum_2ESUC\ V0n))\ c_2Enum_2E0)))) \quad (16)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (((ap\ (ap\ c_2Earithmetic_2E_2D\ c_2Enum_2E0)\ V0m) = c_2Enum_2E0) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2D\ V0m)\ c_2Enum_2E0) = V0m))) \quad (17)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ c_2Enum_2ESUC\ V0n))\ (ap\ c_2Enum_2ESUC\ V1m)) = (ap\ (ap\ c_2Earithmetic_2E_2D\ V0n)\ V1m)))) \quad (18)$$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (22)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\
& p \ V0t))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (((ap \ (c_2Elist_2ELENGTH \ A_27a) \\
& (c_2Elist_2ENIL \ A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a.(\\
& \forall V1t \in (ty_2Elist_2Elist \ A_27a).(ap \ (c_2Elist_2ELENGTH \\
& A_27a) \ (ap \ (ap \ (c_2Elist_2ECONS \ A_27a) \ V0h) \ V1t)) = (ap \ c_2Enum_2ESUC \\
& (ap \ (c_2Elist_2ELENGTH \ A_27a) \ V1t))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1l \in \\
& (ty_2Elist_2Elist \ A_27a).(ap \ (c_2Elist_2ELENGTH \ A_27a) \ (ap \\
& (ap \ (c_2Elist_2ESNOC \ A_27a) \ V0x) \ V1l)) = (ap \ c_2Enum_2ESUC \ (ap \ (\\
& c_2Elist_2ELENGTH \ A_27a) \ V1l))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist \ A_27a)}). \\
& (((p \ (ap \ V0P \ (c_2Elist_2ENIL \ A_27a))) \wedge (\forall V1l \in (ty_2Elist_2Elist \\
& A_27a).(p \ (ap \ V0P \ V1l)) \Rightarrow (\forall V2x \in A_27a.(p \ (ap \ V0P \ (ap \ (ap \ (\\
& c_2Elist_2ESNOC \ A_27a) \ V2x) \ V1l)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A_27a).(p \ (ap \ V0P \ V3l))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p \ (ap \ V0P \ c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum.((p \ (ap \ V0P \ V1n)) \Rightarrow (p \ (ap \ V0P \ (ap \ c_2Enum_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p \ (ap \ V0P \ V2n))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0l \in (ty_2Elist_2Elist\ A.27a).((ap\ (ap\ (c_2Erich_list_2ELASTN \\
& A.27a)\ c_2Enum_2E0)\ V0l) = (c_2Elist_2ENIL\ A.27a))) \wedge (\forall V1n \in \\
& ty_2Enum_2Enum.(\forall V2x \in A.27b.(\forall V3l \in (ty_2Elist_2Elist \\
& A.27b).((ap\ (ap\ (c_2Erich_list_2ELASTN\ A.27b)\ (ap\ c_2Enum_2ESUC \\
& V1n))\ (ap\ (ap\ (c_2Elist_2ESNOC\ A.27b)\ V2x)\ V3l)) = (ap\ (ap\ (c_2Elist_2ESNOC \\
& A.27b)\ V2x)\ (ap\ (ap\ (c_2Erich_list_2ELASTN\ A.27b)\ V1n)\ V3l)))))) \\
& \hspace{15em} (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0l \in (ty_2Elist_2Elist\ A.27a).((ap\ (ap\ (c_2Erich_list_2EBUTLASTN \\
& A.27a)\ c_2Enum_2E0)\ V0l) = V0l)) \wedge (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2x \in A.27b.(\forall V3l \in (ty_2Elist_2Elist\ A.27b). \\
& (ap\ (ap\ (c_2Erich_list_2EBUTLASTN\ A.27b)\ (ap\ c_2Enum_2ESUC\ V1n)) \\
& (ap\ (ap\ (c_2Elist_2ESNOC\ A.27b)\ V2x)\ V3l)) = (ap\ (ap\ (c_2Erich_list_2EBUTLASTN \\
& A.27b)\ V1n)\ V3l)))))) \\
& \hspace{15em} (32)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum.(\\
& \forall V1n \in ty_2Enum_2Enum.(\forall V2l \in (ty_2Elist_2Elist \\
& A.27a).(((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ V0m)\ V1n)) \wedge (p\ (ap \\
& (ap\ c_2Earithmic_2E_3C_3D\ V1n)\ (ap\ (c_2Elist_2ELENGTH\ A.27a) \\
& V2l)))) \Rightarrow ((ap\ (ap\ (c_2Erich_list_2EBUTLASTN\ A.27a)\ V0m)\ (ap\ (\\
& ap\ (c_2Erich_list_2ELASTN\ A.27a)\ V1n)\ V2l)) = (ap\ (ap\ (c_2Erich_list_2ELASTN \\
& A.27a)\ (ap\ (ap\ c_2Earithmic_2E_2D\ V1n)\ V0m))\ (ap\ (ap\ (c_2Erich_list_2EBUTLASTN \\
& A.27a)\ V0m)\ V2l))))))
\end{aligned}$$