

thm_2Erich_list_2EBUTLASTN__MAP
(TMW91eCpEP2KsMFVyMX8QBMBVruLcioLVbM)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))) (\lambda V1P \in 2.V1P)))$

Definition 4 We define $c_2Ebool_2E_F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (c_2Enum_2ESUC_REP m))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (5)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (6)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (7)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Elist_2EMAP\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b^{A_27a})}) \quad (8)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (9)$$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ESNOC\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (10)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EREVERSE\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \quad (12)$$

Let $c_2Elist_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EDROP\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 15 We define `c_Erich_list_2EBUTLASTN` to be $\lambda A_{27a} : \iota. \lambda V0n \in ty_2Enum_2Enum. \lambda V1xs$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\ & (p (ap (ap (c_2Earithmetic_2E_3C_3D (ap (c_2Enum_2ESUC V0n)) (ap \\ & c_2Enum_2ESUC V1m))) \Leftrightarrow (p (ap (ap (c_2Earithmetic_2E_3C_3D V0n) \\ & V1m)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (\neg (p (ap (ap (c_2Earithmetic_2E_3C_3D \\ & (ap (c_2Enum_2ESUC V0n)) c_2Enum_2E0)))) \end{aligned} \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ & A_{27a}. (p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}. ((V0x = V0x) \Leftrightarrow \\ & True)) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow (((ap (c_2Elist_2ELENGTH A_{27a}) \\ & (c_2Elist_2ENIL A_{27a})) = c_2Enum_2E0) \wedge (\forall V0h \in A_{27a}. (\\ & \forall V1t \in (ty_2Elist_2Elist A_{27a}). ((ap (c_2Elist_2ELENGTH \\ & A_{27a}) (ap (ap (c_2Elist_2ECONS A_{27a}) V0h) V1t)) = (ap (c_2Enum_2ESUC \\ & (ap (c_2Elist_2ELENGTH A_{27a}) V1t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0f \in (A.27b^{A.27a}).((ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b) \\
& V0f)\ (c.2Elist.2ENIL\ A.27a)) = (c.2Elist.2ENIL\ A.27b))) \wedge (\forall V1f \in \\
& (A.27b^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in (ty.2Elist.2Elist \\
& A.27a).((ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b)\ V1f)\ (ap\ (ap\ (c.2Elist.2ECONS \\
& A.27a)\ V2h)\ V3t)) = (ap\ (ap\ (c.2Elist.2ECONS\ A.27b)\ (ap\ V1f\ V2h)) \\
& (ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b)\ V1f)\ V3t)))))) \\
& \hspace{15em} (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1l \in \\
& (ty.2Elist.2Elist\ A.27a).((ap\ (c.2Elist.2ELENGTH\ A.27a)\ (ap \\
& (ap\ (c.2Elist.2ESNOC\ A.27a)\ V0x)\ V1l)) = (ap\ c.2Enum.2ESUC\ (ap\ (\\
& c.2Elist.2ELENGTH\ A.27a)\ V1l)))) \\
& \hspace{15em} (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (A.27b^{A.27a}).(\forall V1x \in A.27a.(\forall V2l \in (\\
& ty.2Elist.2Elist\ A.27a).((ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b) \\
& V0f)\ (ap\ (ap\ (c.2Elist.2ESNOC\ A.27a)\ V1x)\ V2l)) = (ap\ (ap\ (c.2Elist.2ESNOC \\
& A.27b)\ (ap\ V0f\ V1x))\ (ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b)\ V0f)\ V2l)))))) \\
& \hspace{15em} (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty.2Elist.2Elist\ A.27a)}). \\
& (((p\ (ap\ V0P\ (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1l \in (ty.2Elist.2Elist \\
& A.27a).((p\ (ap\ V0P\ V1l)) \Rightarrow (\forall V2x \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c.2Elist.2ESNOC\ A.27a)\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\
& A.27a).(p\ (ap\ V0P\ V3l)))))) \\
& \hspace{15em} (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty.2Enum.2Enum}).(((p\ (ap\ V0P\ c.2Enum.2E0)) \wedge \\
& (\forall V1n \in ty.2Enum.2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c.2Enum.2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty.2Enum.2Enum.(p\ (ap\ V0P\ V2n)))))) \\
& \hspace{15em} (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0l \in (ty.2Elist.2Elist\ A.27a).((ap\ (ap\ (c.2Erich_list.2EBUTLASTN \\
& A.27a)\ c.2Enum.2E0)\ V0l) = V0l)) \wedge (\forall V1n \in ty.2Enum.2Enum. \\
& (\forall V2x \in A.27b.(\forall V3l \in (ty.2Elist.2Elist\ A.27b).(\\
& (ap\ (ap\ (c.2Erich_list.2EBUTLASTN\ A.27b)\ (ap\ c.2Enum.2ESUC\ V1n)) \\
& (ap\ (ap\ (c.2Elist.2ESNOC\ A.27b)\ V2x)\ V3l)) = (ap\ (ap\ (c.2Erich_list.2EBUTLASTN \\
& A.27b)\ V1n)\ V3l)))))) \\
& \hspace{15em} (28)
\end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\\ & \quad \forall V0n \in ty_2Enum_2Enum. (\forall V1l \in (ty_2Elist_2Elist \\ & A_{27a}). ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0n)\ (ap\ (c_2Elist_2ELENGTH \\ & A_{27a})\ V1l))) \Rightarrow (\forall V2f \in (A_{27b}^{A_{27a}}). ((ap\ (ap\ (c_2Erich_list_2EBUTLASTN \\ & A_{27b})\ V0n)\ (ap\ (ap\ (c_2Elist_2EMAP\ A_{27a}\ A_{27b})\ V2f)\ V1l)) = (ap \\ & (ap\ (c_2Elist_2EMAP\ A_{27a}\ A_{27b})\ V2f)\ (ap\ (ap\ (c_2Erich_list_2EBUTLASTN \\ & A_{27a})\ V0n)\ V1l)))))) \end{aligned}$$