

thm_2Erich__list_2EELL__CONS
(TMV6PV7dVwipTrtUNeUe2QgG6giiMJXMZsH)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (2)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (3)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELENGTH A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist A_27a)}) \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (5)$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t)))$

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num)$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda P : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40))))$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Definition 12 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda P.\lambda Q. nonempty A \Rightarrow c_2Elist_2ESNOC A \in (((ty_2Elist_2Elist A)^{A})^{(ty_2Elist_2Elist A)^{A}}) \quad (10)$$

Let $c_2Erich_list_2EELL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda P.\lambda Q. nonempty A \Rightarrow c_2Erich_list_2EELL A \in ((A)^{(ty_2Elist_2Elist A)^{A}})^{ty_2Enum_2Enum} \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2ELENGTH\ A_27a) \\ & (c_2Elist_2ENIL\ A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a.(\\ & \forall V1t \in (ty_2Elist_2Elist\ A_27a).(ap\ (c_2Elist_2ELENGTH \\ & A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ c_2Enum_2ESUC \\ & (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a.((ap\ (ap\ (c_2Elist_2ESNOC \\ & A_27a)\ V0x)\ (c_2Elist_2ENIL\ A_27a)) = (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V0x)\ (c_2Elist_2ENIL\ A_27a)))) \wedge (\forall V1x \in A_27a.(\forall V2x.27 \in \\ & A_27a.(\forall V3l \in (ty_2Elist_2Elist\ A_27a).(ap\ (ap\ (c_2Elist_2ESNOC \\ & A_27a)\ V1x)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2x.27)\ V3l)) = (ap\ (\\ & ap\ (c_2Elist_2ECONS\ A_27a)\ V2x.27)\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a) \\ & V1x)\ V3l)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1l \in \\ & (ty_2Elist_2Elist\ A_27a).(ap\ (c_2Elist_2ELENGTH\ A_27a)\ (ap \\ & (ap\ (c_2Elist_2ESNOC\ A_27a)\ V0x)\ V1l)) = (ap\ c_2Enum_2ESUC\ (ap\ (\\ & c_2Elist_2ELENGTH\ A_27a)\ V1l)))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1l \in (ty_2Elist_2Elist \\ & A_27a).(p\ (ap\ V0P\ V1l)) \Rightarrow (\forall V2x \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\ & c_2Elist_2ESNOC\ A_27a)\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a).(p\ (ap\ V0P\ V3l)))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p (ap V0P c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p (ap V0P V2n))))))
\end{aligned} \tag{22}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg(p (ap (ap c_2Eprim_rec_2E_3C V0n) c_2Enum_2E0)))) \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
& (p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Enum_2ESUC V0m)) (ap c_2Enum_2ESUC \\
& V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& A_27a).(\forall V1x \in A_27a.((ap (ap (c_2Erich_list_2EELL A_27a) \\
& c_2Enum_2E0) (ap (ap (c_2Elist_2ESNOC A_27a) V1x) V0l)) = V1x)))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\
& \forall V1x \in A_27a.(\forall V2l \in (ty_2Elist_2Elist A_27a).((\\
& ap (ap (c_2Erich_list_2EELL A_27a) (ap c_2Enum_2ESUC V0n)) (ap \\
& (ap (c_2Elist_2ESNOC A_27a) V1x) V2l)) = (ap (ap (c_2Erich_list_2EELL \\
& A_27a) V0n) V2l))))))
\end{aligned} \tag{26}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\
& \forall V1l \in (ty_2Elist_2Elist A_27a).((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) (ap (c_2Elist_2ELENGTH A_27a) V1l))) \Rightarrow (\forall V2x \in A_27a. \\
& ((ap (ap (c_2Erich_list_2EELL A_27a) V0n) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V2x) V1l)) = (ap (ap (c_2Erich_list_2EELL A_27a) V0n) V1l))))))
\end{aligned}$$