

thm_2Erich_list_2EELL_LENGTH_APPEND
(TMEp6RM2vMhUcA7HW69GxmAD5LYWZvfRtfw)

October 26, 2020

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2ENULL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENULL\ A_27a \in (2^{(ty_2Elist_2Elist\ A_27a)}) \quad (2)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (2^{A_27a}))))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap\ (c_2Ebool_2E_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_2EF))$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (3)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (4)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (5)$$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ESNOC\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})_{A_27a}) \quad (6)$$

Let $c_2Elist_2EFront : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFront\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \quad (7)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 7 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Elist_2ELAST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELAST\ A_27a \in (A_27a)^{(ty_2Elist_2Elist\ A_27a)} \quad (11)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (12)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Erich_list_2EELL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Erich_list_2EELL\ A_27a \in ((A_27a)^{(ty_2Elist_2Elist\ A_27a)})_{ty_2Enum_2Enum} \quad (13)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (14)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})_{(ty_2Elist_2Elist\ A_27a)}) \quad (15)$$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (18) \end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ & p V0t)))))) \quad (20) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (((ap (c_2Elist_2ELENGTH A_27a) \\ & (c_2Elist_2ENIL A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a.(\\ & \forall V1t \in (ty_2Elist_2Elist A_27a).((ap (c_2Elist_2ELENGTH \\ & A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V0h) V1t)) = (ap c_2Enum_2ESUC \\ & (ap (c_2Elist_2ELENGTH A_27a) V1t)))))) \quad (21) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1l \in \\ & (ty_2Elist_2Elist A_27a).((ap (c_2Elist_2ELENGTH A_27a) (ap \\ & (ap (c_2Elist_2ESNOC A_27a) V0x) V1l)) = (ap c_2Enum_2ESUC (ap (\\ & c_2Elist_2ELENGTH A_27a) V1l)))) \quad (22) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1l \in \\ & (ty_2Elist_2Elist A_27a).((ap (c_2Elist_2EFront A_27a) (ap (\\ & (ap (c_2Elist_2ESNOC A_27a) V0x) V1l)) = V1l))) \quad (23) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\ A_27a).(\forall V1x \in A_27a.(\forall V2l2 \in (ty_2Elist_2Elist \\ A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ (ap\ (ap\ (c_2Elist_2ESNOC \\ A_27a)\ V1x)\ V2l2)) = (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V1x)\ (ap\ (ap \\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ V2l2))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1l \in (ty_2Elist_2Elist \\ A_27a).((p\ (ap\ V0P\ V1l)) \Rightarrow (\forall V2x \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\ c_2Elist_2ESNOC\ A_27a)\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ A_27a).(p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\ A_27a).((ap\ (ap\ (c_2Erich_list_2EELL\ A_27a)\ c_2Enum_2E0)\ V0l) = \\ (ap\ (c_2Elist_2ELAST\ A_27a)\ V0l))) \wedge (\forall V1n \in ty_2Enum_2Enum. \\ (\forall V2l \in (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Erich_list_2EELL \\ A_27a)\ (ap\ c_2Enum_2ESUC\ V1n))\ V2l) = (ap\ (ap\ (c_2Erich_list_2EELL \\ A_27a)\ V1n)\ (ap\ (c_2Elist_2EFRONT\ A_27a)\ V2l)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ (\forall V0l \in (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND \\ A_27a)\ V0l)\ (c_2Elist_2ENIL\ A_27a)) = V0l)) \wedge (\forall V1l \in (ty_2Elist_2Elist \\ A_27b).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27b)\ (c_2Elist_2ENIL\ A_27b)) \\ V1l) = V1l))) \end{aligned} \quad (27)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\ A_27a).(\forall V1l2 \in (ty_2Elist_2Elist\ A_27a).((\neg(p\ (ap\ (c_2Elist_2ENULL \\ A_27a)\ V0l1))) \Rightarrow ((ap\ (ap\ (c_2Erich_list_2EELL\ A_27a)\ (ap\ (c_2Elist_2ELENGTH \\ A_27a)\ V1l2))\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ V1l2)) = (\\ ap\ (c_2Elist_2ELAST\ A_27a)\ V0l1)))))) \end{aligned}$$