

thm_Erich_list_EELL_SEG
(TMHw9Xkz2Rskweu9sr3twY8siHGrCYUbNP9)

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Let $ty_Enum_Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_Enum_Enum \tag{1}$$

Let $c_Earithmetic_E_D : \iota$ be given. Assume the following.

$$c_Earithmetic_E_D \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \tag{2}$$

Definition 1 We define $c_Emin_E_D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_Ebool_E_T$ to be $(ap (ap (c_Emin_E_D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_Ebool_E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_Emin_E_D (2^{A-27a}))))$

Definition 4 We define $c_Ebool_E_F$ to be $(ap (c_Ebool_E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_Elist_Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_Elist_Elist\ A0) \tag{3}$$

Let $c_Elist_EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_Elist_EHD\ A.27a \in (A.27a^{(ty_Elist_Elist\ A.27a)}) \tag{4}$$

Let $c_Enum_EREP_num : \iota$ be given. Assume the following.

$$c_Enum_EREP_num \in (omega^{ty_Enum_Enum}) \tag{5}$$

Let $c_Enum_ESUC_REP : \iota$ be given. Assume the following.

$$c_Enum_ESUC_REP \in (omega^{omega}) \tag{6}$$

Let $c_Enum_EABS_num : \iota$ be given. Assume the following.

$$c_Enum_EABS_num \in (ty_Enum_Enum^{omega}) \tag{7}$$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{8}$$

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow Q)$
of type ι .

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Definition 10 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 11 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2E$

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 14 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Elist_2EFRONT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFRONT\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \tag{9}$$

Let $c_2Elist_2ELAST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELAST\ A_27a \in (A_27a)^{(ty_2Elist_2Elist\ A_27a)} \tag{10}$$

Let $c_2Erich_list_2EELL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Erich_list_2EELL\ A_27a \in ((A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum} \tag{11}$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \tag{12}$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \tag{13}$$

Definition 15 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 16 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B V0n))$.

Definition 17 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ESNOC\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (15)$$

Let $c_2Erich_list_2ESEG : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Erich_list_2ESEG\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (16)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (17)$$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$.

Definition 19 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V1n$.

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap (ap c_2Earithmetic_2E_2B\ V1n)\ V0m)))) \quad (18)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2D\ c_2Enum_2E0)\ V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D\ V0m)\ c_2Enum_2E0) = V0m)))) \quad (19)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2D\ (ap\ c_2Enum_2ESUC\ V0n))\ (ap\ c_2Enum_2ESUC\ V1m)) = (ap (ap c_2Earithmetic_2E_2D\ V0n)\ V1m)))) \quad (20)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap\ c_2Enum_2ESUC\ V0m) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \quad (21)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V0n)\ V1m))\ V0n)))) \quad (22)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1m)\ V0n)) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V0n)\ V1m)))))) \quad (23)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ V0m)) \Leftrightarrow ((ap\ c_2Enum_2ESUC\ (ap\ c_2Eprim_rec_2EPRE\ V0m)) = V0m))) \quad (24)$$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h \in A_27a. (\forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2EHD\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = V0h)))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2ELENGTH\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a. (\forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2ELENGTH\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ c_2Enum_2ESUC\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1t))))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2ELENGTH\ A_27a)\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V0x)\ V1l)) = (ap\ c_2Enum_2ESUC\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1l)))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2ELAST\ A_27a)\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V0x)\ V1l)) = V0x)))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2EFront\ A_27a)\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V0x)\ V1l)) = V1l)))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1l \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ V0P\ V1l)) \Rightarrow (\forall V2x \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist\ A_27a). (p\ (ap\ V0P\ V3l)))))) \quad (37)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}). (((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge (\forall V1n \in ty_2Enum_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC\ V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p\ (ap\ V0P\ V2n)))))) \quad (38)$$

Assume the following.

$$(((ap\ c_2Eprim_rec_2EPRE\ c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V0m \in ty_2Enum_2Enum. ((ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Enum_2ESUC\ V0m)) = V0m))) \quad (39)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ c_2Enum_2E0)))) \quad (40)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ (ap\ c_2Enum_2ESUC\ V0m))\ (ap\ c_2Enum_2ESUC\ V1n)))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n)))) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Erich_list_2EELL\ A_27a)\ c_2Enum_2E0)\ V0l) = (ap\ (c_2Elist_2ELAST\ A_27a)\ V0l))) \wedge (\forall V1n \in ty_2Enum_2Enum. (\forall V2l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Erich_list_2EELL\ A_27a)\ (ap\ c_2Enum_2ESUC\ V1n))\ V2l) = (ap\ (ap\ (c_2Erich_list_2EELL\ A_27a)\ V1n)\ (ap\ (c_2Elist_2EFRONT\ A_27a)\ V2l)))))) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist\ A_27a). (\forall V1x \in A_27a. ((ap\ (ap\ (ap\ (c_2Erich_list_2ESEG\ A_27a)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l))\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V1x)\ V0l)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1x)\ (c_2Elist_2ENIL\ A_27a)))))) \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\forall V2l \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m))\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V2l)))) \Rightarrow (\forall V3x \in A_27a. ((ap\ (ap\ (ap\ (c_2Erich_list_2ESEG\ A_27a)\ V0n)\ V1m)\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V3x)\ V2l)) = (ap\ (ap\ (ap\ (c_2Erich_list_2ESEG\ A_27a)\ V0n)\ V1m)\ V2l)))))) \quad (44)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ & \quad \forall V1l \in (ty_2Elist_2Elist\ A_{.27a}).((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ & \quad V0n)\ (ap\ (c_2Elist_2ELENGTH\ A_{.27a})\ V1l))) \Rightarrow ((ap\ (ap\ (c_2Erich_list_2EELL \\ & \quad A_{.27a})\ V0n)\ V1l) = (ap\ (c_2Elist_2EHD\ A_{.27a})\ (ap\ (ap\ (ap\ (c_2Erich_list_2ESEG \\ & \quad A_{.27a})\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ & \quad c_2Earithmetic_2EZERO))))\ (ap\ c_2Eprim_rec_2EPRE\ (ap\ (ap\ c_2Earithmetic_2E_2D \\ & \quad (ap\ (c_2Elist_2ELENGTH\ A_{.27a})\ V1l))\ V0n)))\ V1l)))))) \end{aligned}$$