

thm_2Erich__list_2EEL__APPEND1
(TMV85C7hfBDvBRfNL4F4kcNdon6Ff3SWAAa)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2EEnum_2EEnum : \iota$ be given. Assume the following.

$$nonempty\ ty_2EEnum_2EEnum \tag{1}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2EEnum_2EEnum^{ty_2EEnum_2EEnum})^{ty_2EEnum_2EEnum}) \tag{2}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{3}$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2EEnum_2EEnum^{(ty_2Elist_2Elist\ A_27a)}) \tag{4}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2EEnum_2EEnum}) \tag{5}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{omega}) \quad (6)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (7)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (8)$$

Let $c_2Elist_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEL\ A_27a \in ((A_27a^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \quad (9)$$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (16)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (17)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A_{27a}.(\forall V3x_{27} \in A_{27a}.(\forall V4y \in A_{27a}.(\forall V5y_{27} \in A_{27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{27})) \wedge ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{27})))))) \Rightarrow ((ap (ap (ap (c_{2Ebool_2ECOND} A_{27a}) V0P) V2x) V4y) = (ap (ap (ap (c_{2Ebool_2ECOND} A_{27a}) V1Q) V3x_{27}) V5y_{27})))))) \quad (18)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow ((\forall V0t1 \in A_{27a}.(\forall V1t2 \in A_{27a}.((ap (ap (ap (c_{2Ebool_2ECOND} A_{27a}) c_{2Ebool_2ET} V0t1) V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{27a}.(\forall V3t2 \in A_{27a}.((ap (ap (ap (c_{2Ebool_2ECOND} A_{27a}) c_{2Ebool_2EF} V2t1) V3t2) = V3t2)))))) \quad (19)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0l1 \in (ty_{2Elist_2Elist} A_{27a}).(\forall V1l2 \in (ty_{2Elist_2Elist} A_{27a}).(\forall V2n \in ty_{2Enum_2Enum}.((ap (ap (c_{2Elist_2EEL} A_{27a}) V2n) (ap (ap (c_{2Elist_2EAPPEND} A_{27a}) V0l1) V1l2)) = (ap (ap (ap (c_{2Ebool_2ECOND} A_{27a}) (ap (ap (c_{2Eprim_rec_2E_3C} V2n) (ap (c_{2Elist_2ELENGTH} A_{27a}) V0l1))) (ap (ap (c_{2Elist_2EEL} A_{27a}) V2n) V0l1)) (ap (ap (c_{2Elist_2EEL} A_{27a}) (ap (ap (c_{2Earithmetic_2E_2D} V2n) (ap (c_{2Elist_2ELENGTH} A_{27a}) V0l1))) V1l2)))))) \quad (20)$$

Theorem 1

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0n \in ty_{2Enum_2Enum}.(\forall V1l1 \in (ty_{2Elist_2Elist} A_{27a}).(\forall V2l2 \in (ty_{2Elist_2Elist} A_{27a}).((p (ap (ap (c_{2Eprim_rec_2E_3C} V0n) (ap (c_{2Elist_2ELENGTH} A_{27a}) V1l1))) \Rightarrow ((ap (ap (c_{2Elist_2EEL} A_{27a}) V0n) (ap (ap (c_{2Elist_2EAPPEND} A_{27a}) V1l1) V2l2)) = (ap (ap (c_{2Elist_2EEL} A_{27a}) V0n) V1l1)))))) \quad (20)$$