

thm_2Erich_list_2EEL_FRONT
(TMRAjw4LVDanXoDq7ftogA1RcVAF6cARfy9)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_21` to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{27a}})) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let `c_2Elist_2ENULL` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow c_2Elist_2ENULL A_{27a} \in (2^{(ty_2Elist_2Elist A_{27a})}) \quad (2)$$

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define `c_2Ebool_2E_3F` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap V0P (ap (c_2Emin_2E_40 A_{27a}) P))$

Definition 8 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let `c_2Elist_2EHD` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow c_2Elist_2EHD A_{27a} \in (A_{27a}^{(ty_2Elist_2Elist A_{27a})}) \quad (3)$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (4)$$

Let $c_2Elist_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEL\ A_27a \in ((A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum} \quad (5)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (6)$$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (7)$$

Let $c_2Elist_2EFRONT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFRONT\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \quad (8)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum)^{(ty_2Elist_2Elist\ A_27a)} \quad (9)$$

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (12)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Definition 12 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 13 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum.(V0m = (ap\ c_2Enum_2ESUC\ V1n)))))) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ (ap\ c_2Enum_2ESUC\ V0m))\ (ap\ c_2Enum_2ESUC\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n)))) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (21)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & (p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h \in A_27a. (\forall V1t \in \\ & (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2EHD\ A_27a)\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = V0h))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2ELENGTH\ A_27a) \\ & (c_2Elist_2ENIL\ A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a. (\\ & \forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2ELENGTH\ \\ & A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ c_2Enum_2ESUC \\ & (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((p\ (ap\ (c_2Elist_2ENULL\ A_27a) \\ & (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V0h \in A_27a. (\forall V1t \in (\\ & ty_2Elist_2Elist\ A_27a). (\neg(p\ (ap\ (c_2Elist_2ENULL\ A_27a)\ (ap \\ & (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ & A.27a).((V0l = (c_2Elist_2ENIL\ A.27a)) \vee (\exists V1h \in A.27a.(\exists V2t \in (ty_2Elist_2Elist\ A.27a).(V0l = (ap\ (ap\ (c_2Elist_2ECONS \\ & A.27a)\ V1h)\ V2t))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0n \in ty_2Enum_2Enum.(\forall V1l \in A.27b.(\forall V2ls \in \\ & (ty_2Elist_2Elist\ A.27b).(((ap\ (c_2Elist_2EEL\ A.27a)\ c_2Enum_2E0) = \\ & (c_2Elist_2EHD\ A.27a)) \wedge ((ap\ (ap\ (c_2Elist_2EEL\ A.27b)\ (ap\ c_2Enum_2ESUC \\ & V0n))\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27b)\ V1l)\ V2ls)) = (ap\ (ap\ (c_2Elist_2EEL \\ & A.27b)\ V0n)\ V2ls)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0x \in A.27a.((ap\ (c_2Elist_2EFront \\ & A.27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V0x)\ (c_2Elist_2ENIL\ A.27a))) = \\ & (c_2Elist_2ENIL\ A.27a))) \wedge (\forall V1x \in A.27a.(\forall V2y \in A.27a. \\ & (\forall V3z \in (ty_2Elist_2Elist\ A.27a).((ap\ (c_2Elist_2EFront \\ & A.27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V1x)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & A.27a)\ V2y)\ V3z))) = (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V1x)\ (ap\ (c_2Elist_2EFront \\ & A.27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V2y)\ V3z))))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1xs \in \\ & (ty_2Elist_2Elist\ A.27a).((ap\ (c_2Elist_2ELENGTH\ A.27a)\ (ap \\ & (c_2Elist_2EFront\ A.27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V0x) \\ & V1xs))) = (ap\ (c_2Elist_2ELENGTH\ A.27a)\ V1xs)))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ V0n)\ c_2Enum_2E0)))) \quad (36)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0) \\ (ap\ c_2Enum_2ESUC\ V0n)))) \quad (37)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ & A.27a).(\forall V1n \in ty_2Enum_2Enum.(((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ & V1n)\ (ap\ (c_2Elist_2ELENGTH\ A.27a)\ (ap\ (c_2Elist_2EFront\ A.27a) \\ & V0l)))) \wedge (\neg(p\ (ap\ (c_2Elist_2ENULL\ A.27a)\ V0l)))) \Rightarrow ((ap\ (ap\ (c_2Elist_2EEL \\ & A.27a)\ V1n)\ (ap\ (c_2Elist_2EFront\ A.27a)\ V0l)) = (ap\ (ap\ (c_2Elist_2EEL \\ & A.27a)\ V1n)\ V0l)))))) \end{aligned}$$