

thm_2Erich__list_2EEL__MEM
(TMY73Sy1ctqJw3RNGPLrzyX3hTrdNDdULTf)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{2}$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \tag{3}$$

Let $c_2Elist_2ETL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ETL\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \tag{4}$$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EHD\ A_27a \in (A_27a^{(ty_2Elist_2Elist\ A_27a)}) \tag{5}$$

Let $c_2Elist_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEL\ A_27a \in ((A_27a^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)(ty_2Elist_2Elist A_27a))A_27a) \quad (7)$$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (8)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELIST_TO_SET A_27a \in ((2^{A_27a})(ty_2Elist_2Elist A_27a)) \quad (9)$$

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 9 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_5C$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (13)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 14 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p \ V0t) \Leftrightarrow (p \ V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0h \in A_27a.(\forall V1t \in (ty_2Elist_2Elist \ A_27a).((ap \ (c_2Elist_2EHD \ A_27a) \ (ap \ (ap \ (c_2Elist_2ECONS \ A_27a) \ V0h) \ V1t)) = V0h))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0h \in A_27a.(\forall V1t \in (ty_2Elist_2Elist \ A_27a).((ap \ (c_2Elist_2ETL \ A_27a) \ (ap \ (ap \ (c_2Elist_2ECONS \ A_27a) \ V0h) \ V1t)) = V1t))) \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (((ap\ (c_2Elist_2ELENGTH\ A_27a) \\ & (c_2Elist_2ENIL\ A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a. (\\ & \forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2ELENGTH \\ & A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ c_2Enum_2ESUC \\ & (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0l \in (ty_2Elist_2Elist \\ & A_27a). ((ap\ (ap\ (c_2Elist_2EEL\ A_27a)\ c_2Enum_2E0)\ V0l) = (ap\ (\\ & c_2Elist_2EHD\ A_27a)\ V0l))) \wedge (\forall V1l \in (ty_2Elist_2Elist \\ & A_27a). (\forall V2n \in ty_2Enum_2Enum. ((ap\ (ap\ (c_2Elist_2EEL \\ & A_27a)\ (ap\ c_2Enum_2ESUC\ V2n))\ V1l) = (ap\ (ap\ (c_2Elist_2EEL\ A_27a) \\ & V2n)\ (ap\ (c_2Elist_2ETL\ A_27a)\ V1l)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0x \in A_27a. ((p\ (ap\ (ap \\ & (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a) \\ & (c_2Elist_2ENIL\ A_27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A_27a. (\forall V2h \in \\ & A_27a. (\forall V3t \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V1x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V1x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ V3t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} (\forall V0P \in (2^{ty_2Enum_2Enum}). (((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\ & (\forall V1n \in ty_2Enum_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p\ (ap\ V0P\ V2n)))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} (\forall V0n \in ty_2Enum_2Enum. (\neg (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ & V0n)\ c_2Enum_2E0)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & (p (ap (ap (ap c_2Eprim_rec_2E_3C (ap c_2Enum_2ESUC V0m)) (ap c_2Enum_2ESUC \\
 & V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))))))
 \end{aligned} \tag{30}$$

Theorem 1

$$\begin{aligned}
 & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\
 & \forall V1l \in (ty_2Elist_2Elist A_27a). ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & V0n) (ap (c_2Elist_2ELENGTH A_27a) V1l))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN \\
 & A_27a) (ap (ap (c_2Elist_2EEL A_27a) V0n) V1l)) (ap (c_2Elist_2ELIST_TO_SET \\
 & A_27a) V1l))))))
 \end{aligned}$$