

# thm\_2Erich\_\_list\_2EEL\_\_PRE\_\_LENGTH (TMS1J94zKyTgCxuBN2ckrwLbt7JViktmqRr)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ELAST : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ELAST A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist A\_27a)}) \quad (2)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (3)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ELENGTH A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist A\_27a)}) \quad (4)$$

Let  $c\_2Elist\_2EEL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EEL A\_27a \in ((A\_27a^{(ty\_2Elist\_2Elist A\_27a)})^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ESNOC\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})_{A\_27a}) \quad (6)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (7)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (10)$$

**Definition 7** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (11)$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 9** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.$  **if**  $(\exists x \in A.p\ (ap\ P\ x))$  **then** *(the  $(\lambda x.x \in A \wedge p)$  of type  $\iota \Rightarrow \iota$ ).*

**Definition 10** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ t2))\ t1)$

**Definition 11** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A.\lambda a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda V2t2 \in A.\lambda V3t3 \in A.(ap\ a\ t1\ t2\ t3)))$

**Definition 12** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Ebool\_2E2F\_5C\ m)\ m)\ m)\ m)$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (14) \end{aligned}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1l \in \\ & (ty\_2Elist\_2Elist A\_27a).((ap (c\_2Elist\_2ELENGTH A\_27a) (ap \\ & (ap (c\_2Elist\_2ESNOC A\_27a) V0x) V1l)) = (ap c\_2Enum\_2ESUC (ap ( \\ & c\_2Elist\_2ELENGTH A\_27a) V1l)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1l \in \\ & (ty\_2Elist\_2Elist A\_27a).((ap (c\_2Elist\_2ELAST A\_27a) (ap (ap \\ & (c\_2Elist\_2ESNOC A\_27a) V0x) V1l)) = V0x))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ & A\_27a).(\forall V1x \in A\_27a.((ap (ap (c\_2Elist\_2EEL A\_27a) (ap \\ & (c\_2Elist\_2ELENGTH A\_27a) V0l)) (ap (ap (c\_2Elist\_2ESNOC A\_27a) \\ & V1x) V0l)) = V1x))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist A\_27a)}). \\ & (((p (ap V0P (c\_2Elist\_2ENIL A\_27a))) \wedge (\forall V1l \in (ty\_2Elist\_2Elist \\ & A\_27a).((p (ap V0P V1l)) \Rightarrow (\forall V2x \in A\_27a.(p (ap V0P (ap ( \\ & c\_2Elist\_2ESNOC A\_27a) V2x) V1l))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ & A\_27a).(p (ap V0P V3l)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge (\forall V0m \in \\ & ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Enum\_2ESUC V0m)) = \\ & V0m))) \end{aligned} \quad (21)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ & A\_27a).((\neg(V0l = (c\_2Elist\_2ENIL A\_27a))) \Rightarrow ((ap (ap (c\_2Elist\_2EEL \\ & A\_27a) (ap c\_2Eprim\_rec\_2EPRE (ap (c\_2Elist\_2ELENGTH A\_27a) \\ & V0l))) V0l) = (ap (c\_2Elist\_2ELAST A\_27a) V0l)))) \end{aligned}$$