

thm\_2Erich\_list\_2EEVERY2\_\_REVERSE1  
(TMXQK-  
BEP7y1QnnM3nu4ZrGwJJzftgSvAo3d)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o(x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o(p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND A\_27a A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \tag{2}$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \tag{3}$$

**Definition 7** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27b})$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (4)$$

Let  $c\_2Elist\_2ELIST\_REL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b \in (((2^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27b})^{A\_27a}}) \quad (5)$$

Let  $c\_2Elist\_2EEVERY : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEVERY\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \quad (6)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (7)$$

**Definition 9** We define  $c\_2Epair\_2E2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let  $c\_2Elist\_2EZIP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EZIP\ A\_27a\ A\_27b \in ((ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))^{(ty\_2Epair\_2Eprod\ (ty\_2Elist\_2Elist\ A\_27a\ A\_27b))}) \quad (8)$$

Let  $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (9)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (10)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ & A\_27a). ((ap\ (c\_2Elist\_2EREVERSE\ A\_27a)\ (ap\ (c\_2Elist\_2EREVERSE \\ & A\_27a)\ V0l)) = V0l)) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ & A\_27a). ((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (c\_2Elist\_2EREVERSE \\ & A\_27a)\ V0l)) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V1l2 \in (ty\_2Elist\_2Elist \\ & A\_27b). (\forall V2f \in ((2^{A\_27b})^{A\_27a}). ((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL \\ & A\_27a\ A\_27b)\ V2f)\ V0l1)\ V1l2)) \Leftrightarrow (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a) \\ & V0l1) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27b)\ V1l2)) \wedge (p\ (ap\ (ap\ (c\_2Elist\_2EVERY \\ & (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (c\_2Epair\_2EUNCURRY\ A\_27a \\ & A\_27b\ 2)\ V2f))\ (ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ & (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ V0l1)\ V1l2))))))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1l \in \\ (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (c\_2Elist\_2EVERY\ A\_27a) \\ V0P)\ (ap\ (c\_2Elist\_2EVERSE\ A\_27a)\ V1l))) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EVERY \\ A\_27a)\ V0P)\ V1l)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V1l2 \in (ty\_2Elist\_2Elist \\ A\_27b). (((ap\ (c\_2Elist\_2LENGTH\ A\_27a)\ V0l1) = (ap\ (c\_2Elist\_2LENGTH \\ A\_27b)\ V1l2)) \Rightarrow ((ap\ (c\_2Elist\_2EVERSE\ (ty\_2Epair\_2Eprod\ A\_27a \\ A\_27b))\ (ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ V0l1)\ V1l2)))) = \\ (ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Elist\_2Elist \\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ (ap\ (c\_2Elist\_2EVERSE\ A\_27a) \\ V0l1))\ (ap\ (c\_2Elist\_2EVERSE\ A\_27b)\ V1l2)))))) \end{aligned} \quad (24)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0R \in ((2^{A\_27b})^{A\_27a}). (\forall V1l1 \in (ty\_2Elist\_2Elist \\ A\_27a). (\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27b). ((p\ (ap\ (ap\ (ap \\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b)\ V0R)\ V1l1)\ (ap\ (c\_2Elist\_2EVERSE \\ A\_27b)\ V2l2))) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b) \\ V0R)\ (ap\ (c\_2Elist\_2EVERSE\ A\_27a)\ V1l1))\ V2l2)))))) \end{aligned}$$