

thm\_2Erich\_\_list\_2EEVERY\_\_BUTLASTN  
 (TMXm-  
 phvK5pzQru2wW6Nyb9pFnY9DZYCqFsS)

October 26, 2020

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num ($



**Definition 15** We define  $c\_Erich\_list\_2EBUTLASTN$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1xs$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.( \\ & (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0n)) (ap \\ & c\_2Enum\_2ESUC V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) \\ & V1m)))))) \end{aligned} \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (((ap (c\_2Elist\_2ELENGTH A\_27a) \\ & (c\_2Elist\_2ENIL A\_27a)) = c\_2Enum\_2E0) \wedge (\forall V0h \in A\_27a.( \\ & \forall V1t \in (ty\_2Elist\_2Elist A\_27a).(ap (c\_2Elist\_2ELENGTH \\ & A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V0h) V1t)) = (ap c\_2Enum\_2ESUC \\ & (ap (c\_2Elist\_2ELENGTH A\_27a) V1t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0P \in (2^{A.27a}).((p\ (ap \\
& (ap\ (c.2Elist\_2EEVERY\ A.27a)\ V0P)\ (c.2Elist\_2ENIL\ A.27a))) \Leftrightarrow True)) \wedge \\
& (\forall V1P \in (2^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in (ty\_2Elist\_2Elist \\
& A.27a).((p\ (ap\ (ap\ (c.2Elist\_2EEVERY\ A.27a)\ V1P)\ (ap\ (ap\ (c.2Elist\_2ECONS \\
& A.27a)\ V2h)\ V3t)))) \Leftrightarrow ((p\ (ap\ V1P\ V2h)) \wedge (p\ (ap\ (ap\ (c.2Elist\_2EEVERY \\
& A.27a)\ V1P)\ V3t))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1l \in \\
& (ty\_2Elist\_2Elist\ A.27a).((ap\ (c.2Elist\_2ELENGTH\ A.27a)\ (ap \\
& (ap\ (c.2Elist\_2ESNOC\ A.27a)\ V0x)\ V1l)) = (ap\ c.2Enum\_2ESUC\ (ap\ ( \\
& c.2Elist\_2ELENGTH\ A.27a)\ V1l))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1x \in \\
& A.27a.(\forall V2l \in (ty\_2Elist\_2Elist\ A.27a).((p\ (ap\ (ap\ (c.2Elist\_2EEVERY \\
& A.27a)\ V0P)\ (ap\ (ap\ (c.2Elist\_2ESNOC\ A.27a)\ V1x)\ V2l))) \Leftrightarrow ((p\ (ap \\
& (ap\ (c.2Elist\_2EEVERY\ A.27a)\ V0P)\ V2l)) \wedge (p\ (ap\ V0P\ V1x))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A.27a)}). \\
& (((p\ (ap\ V0P\ (c.2Elist\_2ENIL\ A.27a))) \wedge (\forall V1l \in (ty\_2Elist\_2Elist \\
& A.27a).((p\ (ap\ V0P\ V1l)) \Rightarrow (\forall V2x \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ ( \\
& c.2Elist\_2ESNOC\ A.27a)\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\
& A.27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ c.2Enum\_2E0)) \wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c.2Enum\_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ c.2Eprim\_rec.2E.3C \\
& V0n)\ c.2Enum\_2E0))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Erich\_list\_2EBUTLASTN \\
& \quad A\_27a)\ c\_2Enum\_2E0)\ V0l) = V0l)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V2x \in A\_27b.(\forall V3l \in (ty\_2Elist\_2Elist\ A\_27b). \\
& \quad (ap\ (ap\ (c\_2Erich\_list\_2EBUTLASTN\ A\_27b)\ (ap\ c\_2Enum\_2ESUC\ V1n)) \\
& \quad (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27b)\ V2x)\ V3l)) = (ap\ (ap\ (c\_2Erich\_list\_2EBUTLASTN \\
& \quad \quad A\_27b)\ V1n)\ V3l)))))) \\
& \hspace{15em} (29)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1l \in \\
& \quad (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a) \\
& \quad V0P)\ V1l)) \Rightarrow (\forall V2m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& \quad V2m)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))) \Rightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEVERY \\
& \quad \quad A\_27a)\ V0P)\ (ap\ (ap\ (c\_2Erich\_list\_2EBUTLASTN\ A\_27a)\ V2m)\ V1l))))))
\end{aligned}$$