

# thm\_2Erich\_list\_2EEVERY\_FOLDR\_MAP (TMSADbUyHh585y55isY31oNTeUhkGfYcN8w)

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Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let  $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Elist\_2EMAP\ A.27a\ A.27b \in (((ty\_2Elist\_2Elist\ A.27b)^{(ty\_2Elist\_2Elist\ A.27a)})^{(A.27b^{A.27a})}) \quad (2)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A.27a}))\ (\lambda V1t \in 2.V1t))\ (\lambda V2t \in 2.V2t))\ (\lambda V3t \in 2.V3t))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))\ (\lambda V3t \in 2.V3t))\ (\lambda V4t \in 2.V4t))$

Let  $c\_2Elist\_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Elist\_2EFOLDR\ A.27a\ A.27b \in (((A.27b)^{(ty\_2Elist\_2Elist\ A.27a)})^{(A.27b)^{(A.27b^{A.27a})}}) \quad (3)$$

Let  $c\_2Elist\_2EEVERY : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2EEVERY\ A.27a \in ((2^{(ty\_2Elist\_2Elist\ A.27a)})^{(2^{A.27a})}) \quad (4)$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27b^{A\_27b})^{A\_27a}). (\forall V1e \in \\ & A\_27b. (\forall V2g \in (A\_27a^{A\_27c}). (\forall V3l \in (ty\_2Elist\_2Elist \\ & A\_27c). ((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V0f)\ V1e)\ (ap \\ & (ap\ (c\_2Elist\_2EMAP\ A\_27c\ A\_27a)\ V2g)\ V3l)) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR \\ & A\_27c\ A\_27b)\ (\lambda V4x \in A\_27c. (\lambda V5y \in A\_27b. (ap\ (ap\ V0f\ (ap\ V2g \\ & V4x))\ V5y))))\ V1e)\ V3l)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1l \in \\ & (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (c\_2Elist\_2EVERY\ A\_27a) \\ & V0P)\ V1l)) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27a\ 2)\ (\lambda V2x \in \\ & A\_27a. (\lambda V3l\_27 \in 2. (ap\ (ap\ c\_2Ebool\_2E\_2F\_5C\ (ap\ V0P\ V2x)) \\ & V3l\_27))))\ c\_2Ebool\_2ET)\ V1l)))))) \end{aligned} \quad (9)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1l \in \\ & (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (c\_2Elist\_2EVERY\ A\_27a) \\ & V0P)\ V1l)) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR\ 2\ 2)\ c\_2Ebool\_2E\_2F\_5C \\ & c\_2Ebool\_2ET)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ 2)\ V0P)\ V1l)))))) \end{aligned}$$