

thm_2Erich__list_2EEVERY__LASTN
(TMGtm44d9CtaR7mPs3bZ1N2Q15boCXcdEX6)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_Earithmic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (5)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (6)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (7)$$

Let $c_2Elist_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EEVERY\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (8)$$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ESNOC\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (9)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (10)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EREVERSE\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \quad (12)$$

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ETAKE\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 15 We define $c_Erich_list_2ELASTN$ to be $\lambda A.27a : \iota.\lambda V0n \in ty_2Enum_2Enum.\lambda V1xs \in (ty$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(\\ & (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0n)) (ap \\ & c_2Enum_2ESUC V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0n) \\ & V1m)))))) \end{aligned} \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (((ap (c_2Elist_2ELENGTH A.27a) \\ & (c_2Elist_2ENIL A.27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A.27a.(\\ & \forall V1t \in (ty_2Elist_2Elist A.27a).(ap (c_2Elist_2ELENGTH \\ & A.27a) (ap (ap (c_2Elist_2ECONS A.27a) V0h) V1t)) = (ap c_2Enum_2ESUC \\ & (ap (c_2Elist_2ELENGTH A.27a) V1t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0P \in (2^{A.27a}).((p\ (ap \\
& (ap\ (c.2Elist.2EEVERY\ A.27a)\ V0P)\ (c.2Elist.2ENIL\ A.27a))) \Leftrightarrow True)) \wedge \\
& (\forall V1P \in (2^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in (ty.2Elist.2Elist \\
& A.27a).((p\ (ap\ (ap\ (c.2Elist.2EEVERY\ A.27a)\ V1P)\ (ap\ (ap\ (c.2Elist.2ECONS \\
& A.27a)\ V2h)\ V3t))) \Leftrightarrow ((p\ (ap\ V1P\ V2h)) \wedge (p\ (ap\ (ap\ (c.2Elist.2EEVERY \\
& A.27a)\ V1P)\ V3t)))))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1l \in \\
& (ty.2Elist.2Elist\ A.27a).((ap\ (c.2Elist.2ELENGTH\ A.27a)\ (ap \\
& (ap\ (c.2Elist.2ESNOC\ A.27a)\ V0x)\ V1l)) = (ap\ c.2Enum.2ESUC\ (ap\ (\\
& c.2Elist.2ELENGTH\ A.27a)\ V1l))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1x \in \\
& A.27a.(\forall V2l \in (ty.2Elist.2Elist\ A.27a).((p\ (ap\ (ap\ (c.2Elist.2EEVERY \\
& A.27a)\ V0P)\ (ap\ (ap\ (c.2Elist.2ESNOC\ A.27a)\ V1x)\ V2l))) \Leftrightarrow ((p\ (ap \\
& (ap\ (c.2Elist.2EEVERY\ A.27a)\ V0P)\ V2l)) \wedge (p\ (ap\ V0P\ V1x)))))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty.2Elist.2Elist\ A.27a)}). \\
& (((p\ (ap\ V0P\ (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1l \in (ty.2Elist.2Elist \\
& A.27a).((p\ (ap\ V0P\ V1l)) \Rightarrow (\forall V2x \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c.2Elist.2ESNOC\ A.27a)\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\
& A.27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty.2Enum.2Enum}).(((p\ (ap\ V0P\ c.2Enum.2E0)) \wedge \\
& (\forall V1n \in ty.2Enum.2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c.2Enum.2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty.2Enum.2Enum.(p\ (ap\ V0P\ V2n))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty.2Enum.2Enum.(\neg(p\ (ap\ (ap\ c.2Eprim.2rec.2E.3C \\
& V0n)\ c.2Enum.2E0))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\
& (\forall V0l \in (ty_2Elist_2Elist\ A_{.27a}).((ap\ (ap\ (c_2Erich_list_2ELASTN \\
& A_{.27a})\ c_2Enum_2E0)\ V0l) = (c_2Elist_2ENIL\ A_{.27a}))) \wedge (\forall V1n \in \\
& ty_2Enum_2Enum.(\forall V2x \in A_{.27b}.(\forall V3l \in (ty_2Elist_2Elist \\
& A_{.27b}).((ap\ (ap\ (c_2Erich_list_2ELASTN\ A_{.27b})\ (ap\ c_2Enum_2ESUC \\
& V1n))\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_{.27b})\ V2x)\ V3l)) = (ap\ (ap\ (c_2Elist_2ESNOC \\
& A_{.27b})\ V2x)\ (ap\ (ap\ (c_2Erich_list_2ELASTN\ A_{.27b})\ V1n)\ V3l))))))))) \\
& \hspace{15em} (29)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1l \in \\
& (ty_2Elist_2Elist\ A_{.27a}).((p\ (ap\ (ap\ (c_2Elist_2EEVERY\ A_{.27a}) \\
& V0P)\ V1l)) \Rightarrow (\forall V2m \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& V2m)\ (ap\ (c_2Elist_2ELENGTH\ A_{.27a})\ V1l))) \Rightarrow (p\ (ap\ (ap\ (c_2Elist_2EEVERY \\
& A_{.27a})\ V0P)\ (ap\ (ap\ (c_2Erich_list_2ELASTN\ A_{.27a})\ V2m)\ V1l)))))))))
\end{aligned}$$