

thm_2Erich_list_2EEXISTS__BUTLASTN
(TMJLe4VmZ6XHq5aaBcXuqxEqjYqPPTr7dqd)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{4}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{5}$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EREVERSE\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \tag{6}$$

Let $c_2Elist_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EDROP\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \tag{7}$$

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Erich_list_2EBUTLASTN$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.\lambda V1xs \in$

Let $c_2Erich_list_2ESEG : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Erich_list_2ESEG A_27a \in (((ty_2Elist_2Elist A_27a)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Let $c_2Elist_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEXISTS A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (9)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELENGTH A_27a \in (ty_2Enum_2Enum)^{(ty_2Elist_2Elist A_27a)} \quad (10)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (11)$$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (13)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num m)$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 13 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 14 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Definition 15 We define $c_Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_Earithmetic_2E_2B V0m) c_Enum_2E0) = V0m)) \quad (14)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(p (ap (ap c_Earithmetic_2E_3C_3D (ap (ap c_Earithmetic_2E_2D V0n) V1m)) V0n)))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\forall V1l \in (ty_2Elist_2Elist A_27a).((p (ap (ap c_Earithmetic_2E_3C_3D V0n) (ap (c_Elist_2ELENGTH A_27a) V1l))) \Rightarrow ((ap (ap (c_Erich_list_2EBUTLASTN A_27a) V0n) V1l) = (ap (ap (ap (c_Erich_list_2ESEG A_27a) (ap (ap c_Earithmetic_2E_2D (ap (c_Elist_2ELENGTH A_27a) V1l)) V0n)) c_Enum_2E0) V1l)))))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum.(\forall V1k \in ty_2Enum_2Enum.(\forall V2l \in (ty_2Elist_2Elist A_27a).((p (ap (ap c_Earithmetic_2E_3C_3D (ap (ap c_Earithmetic_2E_2B V0m) V1k)) (ap (c_Elist_2ELENGTH A_27a) V2l))) \Rightarrow (\forall V3P \in (2^{A_27a}).((p (ap (ap (c_Elist_2EEXISTS A_27a) V3P) (ap (ap (ap (c_Erich_list_2ESEG A_27a) V0m) V1k) V2l))) \Rightarrow (p (ap (ap (c_Elist_2EEXISTS A_27a) V3P) V2l)))))))))) \quad (17)$$

Theorem 1

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum.(\forall V1l \in (ty_2Elist_2Elist A_27a).((p (ap (ap c_Earithmetic_2E_3C_3D V0m) (ap (c_Elist_2ELENGTH A_27a) V1l))) \Rightarrow (\forall V2P \in (2^{A_27a}).((p (ap (ap (c_Elist_2EEXISTS A_27a) V2P) (ap (ap (c_Erich_list_2EBUTLASTN A_27a) V0m) V1l))) \Rightarrow (p (ap (ap (c_Elist_2EEXISTS A_27a) V2P) V1l))))))))))$$