thm_2Erich__list_2EEXISTS__DROP (TMKkykFmZm8oypqqVGucaDVNwdtScdq1q7y)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (2)$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (3)$$

Let $c\_2Elist\_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EDROP\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (4)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Let $c\_2Erich\_\_list\_2ESEG : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Erich\_\_list\_2ESEG\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (5)$$

Let $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEXISTS\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \quad (6)$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (7)$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

Definition 2 *We define* $\mathsf{c\_2Ebool\_2ET}$ *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* $\mathsf{c\_2Ebool\_2E\_21}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* $\mathsf{c\_2Ebool\_2EF}$ *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* $\mathsf{c\_2Emin\_2E\_3D\_3D\_3E}$ *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* $\mathsf{c\_2Ebool\_2E\_7E}$ *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* $\mathsf{c\_2Ebool\_2E\_2F\_5C}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{10}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{11}$$

Definition 8 *We define* $\mathsf{c\_2Enum\_2ESUC}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* $\mathsf{c\_2Emin\_2E\_40}$ *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 11 *We define* $\mathsf{c\_2Eprim\_\_rec\_2E\_3C}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* $\mathsf{c\_2Ebool\_2E\_5C\_2F}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 13 *We define* $\mathsf{c\_2Earithmetic\_2E\_3C\_3D}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Assume the following.

$$\begin{gathered}(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m)) \Rightarrow ((ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n))\ V1n) = V0m))))\end{gathered} \tag{12}$$

Assume the following.

$$\begin{gathered}(\forall V0m \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\ V0m)\ V0m)))\end{gathered} \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))) \Rightarrow ((ap\ (ap\ (c\_2Elist\_2EDROP\ A\_27a)\ V0n)\ V1l) = (ap\ (ap\ (ap\ (c\_2Erich\_\_list\_2ESEG\ A\_27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))\ V0n))\ V0n)\ V1l))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1k \in ty\_2Enum\_2Enum.(\forall V2l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1k))\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V2l))) \Rightarrow (\forall V3P \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V3P)\ (ap\ (ap\ (ap\ (c\_2Erich\_\_list\_2ESEG\ A\_27a)\ V0m)\ V1k)\ V2l))) \Rightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V3P)\ V2l)))))))) \tag{15}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))) \Rightarrow (\forall V2P \in (2^{A\_27a}). ((p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V2P)\ (ap\ (ap\ (c\_2Elist\_2EDROP\ A\_27a)\ V0m)\ V1l))) \Rightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V2P)\ V1l)))))))$$