

thm\_2Erich\_list\_2EEXISTS\_FOLDL\_MAP  
 (TMNnZTtDx-  
 cmZw8uFJe9PxPCC1JUUhQeQhBw)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$ .  
 Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Elist\_2EMAP A\_27a A\_27b \in (((ty\_2Elist\_2Elist A\_27b)^{(ty\_2Elist\_2Elist A\_27a)})^{(A\_27b^{A\_27a})}) \quad (2)$$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Elist\_2EFOLDL A\_27a A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27b})^{((A\_27b^{A\_27a})^{A\_27b})}) \quad (3)$$

Let  $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2EEXISTS A\_27a \in ((2^{(ty\_2Elist\_2Elist A\_27a)})^{(2^{A\_27a})}) \quad (4)$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A.27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (6)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\begin{aligned} &\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ &\quad nonempty\ A.27c \Rightarrow (\forall V0f \in ((A.27a^{A.27b})^{A.27a}).(\forall V1e \in \\ &\quad A.27a.(\forall V2g \in (A.27b^{A.27c}).(\forall V3l \in (ty\_2Elist\_2Elist \\ &\quad A.27c).((ap\ (ap\ (ap\ (c.2Elist\_2EFOLDL\ A.27b\ A.27a)\ V0f)\ V1e)\ (ap \\ &\quad (ap\ (c.2Elist\_2EMAP\ A.27c\ A.27b)\ V2g)\ V3l)) = (ap\ (ap\ (ap\ (c.2Elist\_2EFOLDL \\ &\quad A.27c\ A.27a)\ (\lambda V4x \in A.27a.(\lambda V5y \in A.27c.(ap\ (ap\ V0f\ V4x)\ ( \\ &\quad ap\ V2g\ V5y))))))\ V1e)\ V3l)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} &\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1l \in \\ &\quad (ty\_2Elist\_2Elist\ A.27a).((p\ (ap\ (ap\ (c.2Elist\_2EEXISTS\ A.27a)\ \\ &\quad V0P)\ V1l)) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c.2Elist\_2EFOLDL\ A.27a\ 2)\ (\lambda V2l.27 \in \\ &\quad 2.(\lambda V3x \in A.27a.(ap\ (ap\ c.2Ebool\_2E\_5C\_2F\ V2l.27)\ (ap\ V0P\ V3x)))))) \\ &\quad c.2Ebool\_2EF)\ V1l)))))) \end{aligned} \quad (9)$$

**Theorem 1**

$$\begin{aligned} &\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1l \in \\ &\quad (ty\_2Elist\_2Elist\ A.27a).((p\ (ap\ (ap\ (c.2Elist\_2EEXISTS\ A.27a)\ \\ &\quad V0P)\ V1l)) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c.2Elist\_2EFOLDL\ 2\ 2)\ c.2Ebool\_2E\_5C\_2F \\ &\quad c.2Ebool\_2EF)\ (ap\ (ap\ (c.2Elist\_2EMAP\ A.27a\ 2)\ V0P)\ V1l)))))) \end{aligned}$$