

thm_2Erich_list_2EEXISTS_FOLDR (TMVpz6TKejCUr29wwVaXyEir466anhG9bJr)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EFOLDR \\ A_27a A_27b \in (((A_27b)^{(ty_2Elist_2Elist A_27a)})^{A_27b})^{((A_27b)^{A_27b})^{A_27a}} \end{aligned} \quad (2)$$

Definition 7 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Let $c_2Elist_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEXISTS A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (3)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (5)$$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$
 Assume the following.

$$True \tag{6}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{7}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{8}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \tag{10}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\tag{11}$$

$$\begin{aligned} & (\forall V0f \in ((A_27b^{A_27b})^{A_27a}).(\forall V1e \in A_27b.((ap (\\ & ap (ap (c_2Elist_2EFOLDR A_27a A_27b) V0f) V1e) (c_2Elist_2ENIL \\ & A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b^{A_27b})^{A_27a}).(\forall V3e \in \\ & A_27b.(\forall V4x \in A_27a.(\forall V5l \in (ty_2Elist_2Elist A_27a). \\ & ((ap (ap (ap (c_2Elist_2EFOLDR A_27a A_27b) V2f) V3e) (ap (ap (c_2Elist_2ECONS \\ & A_27a) V4x) V5l)) = (ap (ap V2f V4x) (ap (ap (ap (c_2Elist_2EFOLDR \\ & A_27a A_27b) V2f) V3e) V5l)))))) \end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow ((\forall V0P \in (2^{A_27a}).((p (ap (ap (c_2Elist_2EEXISTS A_27a) V0P) (c_2Elist_2ENIL A_27a))) \Leftrightarrow False)) \wedge (\forall V1P \in (2^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist A_27a).((p (ap (ap (c_2Elist_2EEXISTS A_27a) V1P) (ap (ap (c_2Elist_2ECONS A_27a) V2h) V3t))) \Leftrightarrow ((p (ap V1P V2h)) \vee (p (ap (ap (c_2Elist_2EEXISTS A_27a) V1P) V3t))))))))) \tag{12}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist A_27a)}). \tag{13}$$

$$\begin{aligned} & (((p (ap V0P (c_2Elist_2ENIL A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A_27a.(p (ap V0P (ap (ap (\\ & c_2Elist_2ECONS A_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a).(p (ap V0P V3l)))) \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}). (\forall V1l \in \\ & (ty_2Elist_2Elist \ A_{27a}). ((p \ (ap \ (ap \ (c_2Elist_2EEXISTS \ A_{27a}) \\ & V0P) \ V1l))) \Leftrightarrow (p \ (ap \ (ap \ (ap \ (c_2Elist_2EFOLDR \ A_{27a} \ 2) \ (\lambda V2x \in \\ & A_{27a}. (\lambda V3l.27 \in 2. (ap \ (ap \ c_2Ebool_2E_5C_2F \ (ap \ V0P \ V2x)) \\ & V3l.27)))) \ c_2Ebool_2EF) \ V1l)))))) \end{aligned}$$