

thm_2Erich__list_2EEXISTS__LASTN
 (TMaXzFmzburaboWWZKAJ-
 DYNKD6YZKKoJ5Co)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{2}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{3}$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EREVERSE\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \tag{4}$$

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ETAKE\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \tag{5}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V0x \in 2.V0x)))$

Definition 4 We define $c_2Erich_list_2ELASTN$ to be $\lambda A_27a : \iota. \lambda V0n \in ty_2Enum_2Enum. \lambda V1xs \in (ty_2Enum_2Enum)^n. (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V0x \in 2.V0x)\ (\lambda V1x \in 2.V1x)$

Let $c_2Erich_list_2ESEG : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Erich_list_2ESEG\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (6)$$

Let $c_2Elist_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEXISTS\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A-27a})}) \quad (7)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum)^{(ty_2Elist_2Elist\ A_27a)} \quad (8)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Definition 5 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (12)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A)\ P)$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ P)))$

Definition 12 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V1n$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 14 We define $c_Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap (ap (c_Earithmetic_2E_2B V0m) V1n) = (ap (ap (c_Earithmetic_2E_2B V1n) V0m)))))) \quad (13)$$

Assume the following.

$$(p (ap (ap (c_Earithmetic_2E_3C_3D V1n) V0m)) \Rightarrow ((ap (ap (c_Earithmetic_2E_2B (ap (ap (c_Earithmetic_2E_2D V0m) V1n)) V1n) = V0m)))))) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (p (ap (ap (c_Earithmetic_2E_3C_3D V0m) V0m)))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\forall V1l \in (ty_2Elist_2Elist A_27a). ((p (ap (ap (c_Earithmetic_2E_3C_3D V0n) (ap (c_Elist_2ELENGTH A_27a) V1l)) \Rightarrow ((ap (ap (c_Erich_list_2ELASTN A_27a) V0n) V1l) = (ap (ap (ap (c_Erich_list_2ESEGA A_27a) V0n) (ap (ap (c_Earithmetic_2E_2D (ap (c_Elist_2ELENGTH A_27a) V1l)) V0n)) V1l))))))))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum. (\forall V1k \in ty_2Enum_2Enum. (\forall V2l \in (ty_2Elist_2Elist A_27a). ((p (ap (ap (c_Earithmetic_2E_3C_3D (ap (ap (c_Earithmetic_2E_2B V0m) V1k)) (ap (c_Elist_2ELENGTH A_27a) V2l)) \Rightarrow (\forall V3P \in (2^{A_27a}). ((p (ap (ap (c_Elist_2EEXISTS A_27a) V3P) (ap (ap (ap (c_Erich_list_2ESEGA A_27a) V0m) V1k) V2l)) \Rightarrow (p (ap (ap (c_Elist_2EEXISTS A_27a) V3P) V2l))))))))))))) \quad (17)$$

Theorem 1

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum. (\forall V1l \in (ty_2Elist_2Elist A_27a). ((p (ap (ap (c_Earithmetic_2E_3C_3D V0m) (ap (c_Elist_2ELENGTH A_27a) V1l)) \Rightarrow (\forall V2P \in (2^{A_27a}). ((p (ap (ap (c_Elist_2EEXISTS A_27a) V2P) (ap (ap (c_Erich_list_2ELASTN A_27a) V0m) V1l)) \Rightarrow (p (ap (ap (c_Elist_2EEXISTS A_27a) V2P) V1l)))))))))))))$$