

# thm\_2Erich\_list\_2EEXISTS\_TAKE (TMNkqCE- ViDA4dTPpA9BTZmUqxQJBqoKqkep)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{4}$$

Let  $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETAKE\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{5}$$

Let  $c\_2Erich\_list\_2ESEG : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Erich\_list\_2ESEG\ A\_27a \in (( (ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)} )^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (6)$$

Let  $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEXISTS\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A-27a})}) \quad (7)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum)^{(ty\_2Elist\_2Elist\ A\_27a)} \quad (8)$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (11)$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ ($

**Definition 10** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge$   
of type  $\iota \Rightarrow \iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 12** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 14** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ c\_2Enum\_2E0) = V0m)) \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum. ( \\ & \forall V1l \in (ty\_2Elist\_2Elist\ A_{.27a}). ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\ & V0n)\ (ap\ (c\_2Elist\_2ELENGTH\ A_{.27a})\ V1l))) \Rightarrow ((ap\ (ap\ (c\_2Elist\_2ETAKE \\ & A_{.27a})\ V0n)\ V1l) = (ap\ (ap\ (ap\ (c\_2Erich\_list\_2ESEGE\ A_{.27a})\ V0n) \\ & c\_2Enum\_2E0)\ V1l)))))) \end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum. ( \\ & \forall V1k \in ty\_2Enum\_2Enum. (\forall V2l \in (ty\_2Elist\_2Elist \\ & A_{.27a}). ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ & V0m)\ V1k))\ (ap\ (c\_2Elist\_2ELENGTH\ A_{.27a})\ V2l))) \Rightarrow (\forall V3P \in \\ & (2^{A_{.27a}}). ((p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A_{.27a})\ V3P)\ (ap\ (ap\ (ap \\ & (c\_2Erich\_list\_2ESEGE\ A_{.27a})\ V0m)\ V1k)\ V2l))) \Rightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS \\ & A_{.27a})\ V3P)\ V2l))))))))) \end{aligned} \tag{16}$$

**Theorem 1**

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum. ( \\ & \forall V1l \in (ty\_2Elist\_2Elist\ A_{.27a}). ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\ & V0m)\ (ap\ (c\_2Elist\_2ELENGTH\ A_{.27a})\ V1l))) \Rightarrow (\forall V2P \in (2^{A_{.27a}}). \\ & ((p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A_{.27a})\ V2P)\ (ap\ (ap\ (c\_2Elist\_2ETAKE \\ & A_{.27a})\ V0m)\ V1l))) \Rightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A_{.27a})\ V2P)\ V1l)))))) \end{aligned}$$