

thm_2Erich_list_2EFCOMM_FOLDL_APPEND
(TMbTKCW7tLeJNTWwuunH2kZUMhaUEjvrQ3F)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})$

Definition 4 We define $c_2Ecombin_2EFCOMM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27a^{A_27b})^{A_27c})$

Definition 5 We define $c_2Ecombin_2ERIGHT_ID$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27b})^{A_27a})$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (2)$$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ESNOC A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (3)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EFOLDL A_27a A_27b \in (((A_27b)^{(ty_2Elist_2Elist A_27a)})^{A_27b})^{((A_27b)^{A_27a})^{A_27b}} \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (5)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (6)$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0f \in ((A_27b^{A_27a})^{A_27b}).(\forall V1e \in A_27b.((ap\ (\\ & ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V0f)\ V1e)\ (c_2Elist_2ENIL \\ & A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b^{A_27a})^{A_27b}).(\forall V3e \in \\ & A_27b.(\forall V4x \in A_27a.(\forall V5l \in (ty_2Elist_2Elist\ A_27a). \\ & ((ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V4x)\ V5l)) = (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V2f) \\ & (ap\ (ap\ V2f\ V3e)\ V4x))\ V5l)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\ & A_27a).(\forall V1x \in A_27a.(\forall V2l2 \in (ty_2Elist_2Elist \\ & A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ (ap\ (ap\ (c_2Elist_2ESNOC \\ & A_27a)\ V1x)\ V2l2)) = (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V1x)\ (ap\ (ap \\ & (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ V2l2)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}), \\
& (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1l \in (ty_2Elist_2Elist \\
& A_27a).(p\ (ap\ V0P\ V1l))) \Rightarrow (\forall V2x \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c_2Elist_2ESNOC\ A_27a\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A_27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in ((A_27b^{A_27a})^{A_27b}).(\forall V1e \in A_27b.(\forall V2x \in \\
& A_27a.(\forall V3l \in (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (ap\ (c_2Elist_2EFOLDL \\
& A_27a\ A_27b)\ V0f)\ V1e)\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V2x)\ V3l)) = \\
& (ap\ (ap\ V0f\ (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V0f)\ V1e)\ V3l)) \\
& V2x))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0l \in (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND \\
& A_27a\ V0l)\ (c_2Elist_2ENIL\ A_27a)) = V0l)) \wedge (\forall V1l \in (ty_2Elist_2Elist \\
& A_27b).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27b)\ (c_2Elist_2ENIL\ A_27b)) \\
& V1l) = V1l)))
\end{aligned} \tag{15}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in ((A_27a^{A_27b})^{A_27a}).(\forall V1g \in ((A_27a^{A_27a})^{A_27a}). \\
& ((p\ (ap\ (ap\ (c_2Ecombin_2EFCOMM\ A_27a\ A_27b\ A_27a)\ V0f)\ V1g))) \Rightarrow (\\
& \forall V2e \in A_27a.((p\ (ap\ (ap\ (c_2Ecombin_2ERIGHT_ID\ A_27a\ A_27a) \\
& V1g)\ V2e))) \Rightarrow (\forall V3l1 \in (ty_2Elist_2Elist\ A_27b).(\forall V4l2 \in \\
& (ty_2Elist_2Elist\ A_27b).((ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27b \\
& A_27a)\ V0f)\ V2e)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27b)\ V3l1)\ V4l2))) = \\
& (ap\ (ap\ V1g\ (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27b\ A_27a)\ V0f)\ V2e)\ V3l1)) \\
& (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27b\ A_27a)\ V0f)\ V2e)\ V4l2))))))
\end{aligned}$$