

thm_2Erich_list_2EFILTER_FLAT (TMHbi6THEF2cbqczjba7hACkCQNPZjni6zA)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EFLAT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EFLAT A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist (ty_2Elist_2Elist A_27a))}) \quad (2)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EMAP A_27a A_27b \in (((ty_2Elist_2Elist A_27b)^{(ty_2Elist_2Elist A_27a)})^{(A_27b^{A_27a})}) \quad (3)$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap P x))$ then $(the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Emin_2E_40 (A_27a) (ap P x))$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (5)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (6)$$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFILTER\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (7)$$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (((ap\ (c_2Elist_2EFLAT\ A_27a)\ (\\ & c_2Elist_2ENIL\ (ty_2Elist_2Elist\ A_27a))) = (c_2Elist_2ENIL \\ & A_27a)) \wedge (\forall V0h \in (ty_2Elist_2Elist\ A_27a).(\forall V1t \in \\ & (ty_2Elist_2Elist\ (ty_2Elist_2Elist\ A_27a)).((ap\ (c_2Elist_2EFLAT \\ & A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Elist_2Elist\ A_27a))\ V0h) \\ & V1t)) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0h)\ (ap\ (c_2Elist_2EFLAT \\ & A_27a)\ V1t)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0f \in (A.27b^{A.27a}).((ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b) \\
& V0f)\ (c.2Elist.2ENIL\ A.27a)) = (c.2Elist.2ENIL\ A.27b))) \wedge (\forall V1f \in \\
& (A.27b^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in (ty.2Elist.2Elist \\
& A.27a).((ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b)\ V1f)\ (ap\ (ap\ (c.2Elist.2ECONS \\
& A.27a)\ V2h)\ V3t)) = (ap\ (ap\ (c.2Elist.2ECONS\ A.27b)\ (ap\ V1f\ V2h)) \\
& (ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b)\ V1f)\ V3t)))))) \\
& \hspace{15em} (12)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0P \in (2^{A.27a}).((ap\ (\\
& ap\ (c.2Elist.2EFILTER\ A.27a)\ V0P)\ (c.2Elist.2ENIL\ A.27a)) = (c.2Elist.2ENIL \\
& A.27a))) \wedge (\forall V1P \in (2^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in \\
& (ty.2Elist.2Elist\ A.27a).((ap\ (ap\ (c.2Elist.2EFILTER\ A.27a) \\
& V1P)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a)\ V2h)\ V3t)) = (ap\ (ap\ (ap\ (c.2Ebool.2ECOND \\
& (ty.2Elist.2Elist\ A.27a))\ (ap\ V1P\ V2h))\ (ap\ (ap\ (c.2Elist.2ECONS \\
& A.27a)\ V2h)\ (ap\ (ap\ (c.2Elist.2EFILTER\ A.27a)\ V1P)\ V3t))))\ (ap\ (ap \\
& (c.2Elist.2EFILTER\ A.27a)\ V1P)\ V3t)))))) \\
& \hspace{15em} (13)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty.2Elist.2Elist\ A.27a)}). \\
& (((p\ (ap\ V0P\ (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\
& A.27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c.2Elist.2ECONS\ A.27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\
& A.27a).(p\ (ap\ V0P\ V3l)))))) \\
& \hspace{15em} (14)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1L \in \\
& (ty.2Elist.2Elist\ A.27a).(\forall V2M \in (ty.2Elist.2Elist\ A.27a). \\
& ((ap\ (ap\ (c.2Elist.2EFILTER\ A.27a)\ V0P)\ (ap\ (ap\ (c.2Elist.2EAPPEND \\
& A.27a)\ V1L)\ V2M)) = (ap\ (ap\ (c.2Elist.2EAPPEND\ A.27a)\ (ap\ (ap\ (c.2Elist.2EFILTER \\
& A.27a)\ V0P)\ V1L))\ (ap\ (ap\ (c.2Elist.2EFILTER\ A.27a)\ V0P)\ V2M)))))) \\
& \hspace{15em} (15)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1l \in \\
& (ty.2Elist.2Elist\ (ty.2Elist.2Elist\ A.27a)).((ap\ (ap\ (c.2Elist.2EFILTER \\
& A.27a)\ V0P)\ (ap\ (c.2Elist.2EFLAT\ A.27a)\ V1l)) = (ap\ (c.2Elist.2EFLAT \\
& A.27a)\ (ap\ (ap\ (c.2Elist.2EMAP\ (ty.2Elist.2Elist\ A.27a)\ (ty.2Elist.2Elist \\
& A.27a))\ (ap\ (c.2Elist.2EFILTER\ A.27a)\ V0P))\ V1l))))))
\end{aligned}$$