

thm\_2Erich\_list\_2EFILTER\_FOLDL  
 (TMWnZmjJzhAAWCh-  
 FgcNs9w4jBHS2BWCQkxV)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Elist\_2ECONS A.27a \in (((ty\_2Elist\_2Elist A.27a)^{(ty\_2Elist\_2Elist A.27a)})^{A.27a}) \quad (2)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Elist\_2ENIL A.27a \in (ty\_2Elist\_2Elist A.27a) \quad (3)$$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t)))$

Let  $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c\_2Elist\_2EFOLDL A.27a A.27b \in (((A.27b)^{(ty\_2Elist\_2Elist A.27a)})^{A.27b})^{((A.27b)^{A-27a})^{A-27b}} \quad (4)$$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 7** We define  $c\_2Emin\_2E.40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 8** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.(a$

Let  $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Elist\_2ESNOC A.27a \in (((ty\_2Elist\_2Elist A.27a)^{(ty\_2Elist\_2Elist A.27a)})^{A.27a}) \quad (5)$$

Let  $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Elist\_2EFILTER A.27a \in (((ty\_2Elist\_2Elist A.27a)^{(ty\_2Elist\_2Elist A.27a)})^{(2^{A.27a})}) \quad (6)$$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A.27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (8)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0P \in (2^{A.27a}).((ap ( \\ & ap (c\_2Elist\_2EFILTER A.27a) V0P) (c\_2Elist\_2ENIL A.27a)) = (c\_2Elist\_2ENIL \\ & A.27a))) \wedge (\forall V1P \in (2^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in \\ & (ty\_2Elist\_2Elist A.27a).((ap (ap (c\_2Elist\_2EFILTER A.27a) \\ & V1P) (ap (ap (c\_2Elist\_2ECONS A.27a) V2h) V3t)) = (ap (ap (ap (c\_2Ebool\_2ECOND \\ & (ty\_2Elist\_2Elist A.27a) (ap V1P V2h)) (ap (ap (c\_2Elist\_2ECONS \\ & A.27a) V2h) (ap (ap (c\_2Elist\_2EFILTER A.27a) V1P) V3t)))) (ap (ap \\ & (c\_2Elist\_2EFILTER A.27a) V1P) V3t)))))) \quad (10) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ & (\forall V0f \in ((A.27b^{A.27a})^{A.27b}).(\forall V1e \in A.27b.((ap ( \\ & ap (ap (c\_2Elist\_2EFOLDL A.27a A.27b) V0f) V1e) (c\_2Elist\_2ENIL \\ & A.27a)) = V1e))) \wedge (\forall V2f \in ((A.27b^{A.27a})^{A.27b}).(\forall V3e \in \\ & A.27b.(\forall V4x \in A.27a.(\forall V5l \in (ty\_2Elist\_2Elist A.27a). \\ & ((ap (ap (ap (c\_2Elist\_2EFOLDL A.27a A.27b) V2f) V3e) (ap (ap (c\_2Elist\_2ECONS \\ & A.27a) V4x) V5l)) = (ap (ap (ap (c\_2Elist\_2EFOLDL A.27a A.27b) V2f) \\ & (ap (ap V2f V3e) V4x)) V5l)))))) \quad (11) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}), \\
& (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1l \in (ty\_2Elist\_2Elist \\
& A\_27a).(p\ (ap\ V0P\ V1l))) \Rightarrow (\forall V2x \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ ( \\
& c\_2Elist\_2ESNOC\ A\_27a\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\
& A\_27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0f \in ((A\_27b^{A\_27a})^{A\_27b}).(\forall V1e \in A\_27b.(\forall V2x \in \\
& A\_27a.(\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL \\
& A\_27a\ A\_27b)\ V0f)\ V1e)\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V2x)\ V3l)) = \\
& (ap\ (ap\ V0f\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V0f)\ V1e)\ V3l)) \\
& V2x))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1x \in \\
& A\_27a.(\forall V2l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EFILTER \\
& A\_27a)\ V0P)\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V1x)\ V2l)) = (ap\ (ap\ ( \\
& ap\ (c\_2Ebool\_2ECOND\ (ty\_2Elist\_2Elist\ A\_27a))\ (ap\ V0P\ V1x))\ (ap \\
& (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V1x)\ (ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a) \\
& V0P)\ V2l)))\ (ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a)\ V0P)\ V2l))))))
\end{aligned} \tag{14}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1l \in \\
& (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a) \\
& V0P)\ V1l) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ (ty\_2Elist\_2Elist \\
& A\_27a))\ (\lambda V2l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\lambda V3x \in A\_27a. \\
& (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Elist\_2Elist\ A\_27a))\ (ap\ V0P \\
& V3x))\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V3x)\ V2l\_27))\ V2l\_27))) \\
& (c\_2Elist\_2ENIL\ A\_27a))\ V1l))))))
\end{aligned}$$