

thm_2Erich__list_2EFOLDL__FOLDR__REVERSE (TMad7efkD5EFtaL4ctxNqpoDJypdHLydr9U)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EFOLDR A_27a A_27b \in (((A_27b^{(ty_2Elist_2Elist A_27a)})^{A_27b})^{(A_27b^{A_27b})^{A_27a}}) \quad (2)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (3)$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EREVERSE A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)}) \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (5)$$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ESNOC A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})_{A_27a}) \quad (6)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Elist_2EFOLDL A_27a A_27b \in (((A_27b)^{(ty_2Elist_2Elist A_27a)})_{A_27b})^{((A_27b)^{A_27a})^{A_27b}} \quad (7)$$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (9)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\\ & \quad (\forall V0f \in ((A_27b)^{A_27b})^{A_27a}. (\forall V1e \in A_27b. ((ap (\\ & \quad ap (ap (c_2Elist_2EFOLDL A_27a A_27b) V0f) V1e) (c_2Elist_2ENIL \\ & \quad A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b)^{A_27b})^{A_27a}. (\forall V3e \in \\ & \quad A_27b. (\forall V4x \in A_27a. (\forall V5l \in (ty_2Elist_2Elist A_27a). \\ & \quad ((ap (ap (ap (c_2Elist_2EFOLDL A_27a A_27b) V2f) V3e) (ap (ap (c_2Elist_2ECONS \\ & \quad A_27a) V4x) V5l)) = (ap (ap (ap (c_2Elist_2EFOLDL A_27a A_27b) V2f) \\ & \quad A_27a A_27b) V2f) V3e) V5l)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\\ & \quad (\forall V0f \in ((A_27b)^{A_27a})^{A_27b}. (\forall V1e \in A_27b. ((ap (\\ & \quad ap (ap (c_2Elist_2EFOLDL A_27a A_27b) V0f) V1e) (c_2Elist_2ENIL \\ & \quad A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b)^{A_27a})^{A_27b}. (\forall V3e \in \\ & \quad A_27b. (\forall V4x \in A_27a. (\forall V5l \in (ty_2Elist_2Elist A_27a). \\ & \quad ((ap (ap (ap (c_2Elist_2EFOLDL A_27a A_27b) V2f) V3e) (ap (ap (c_2Elist_2ECONS \\ & \quad A_27a) V4x) V5l)) = (ap (ap (ap (c_2Elist_2EFOLDL A_27a A_27b) V2f) \\ & \quad (ap (ap (ap (c_2Elist_2EFOLDL A_27a A_27b) V2f) V3e) V4x)) V5l)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & ((ap\ (c_2Elist_2EVERSE\ A_27b)\ (c_2Elist_2ENIL\ A_27b)) = (c_2Elist_2ENIL \\ & \quad A_27b)) \wedge (\forall V0x \in A_27a. (\forall V1l \in (ty_2Elist_2Elist \\ & \quad A_27a). ((ap\ (c_2Elist_2EVERSE\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & \quad A_27a)\ V0x)\ V1l)) = (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V0x)\ (ap\ (c_2Elist_2EVERSE \\ & \quad A_27a)\ V1l)))))) \end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1l \in \\ & (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2EVERSE\ A_27a)\ (ap \\ & (ap\ (c_2Elist_2ESNOC\ A_27a)\ V0x)\ V1l)) = (ap\ (ap\ (c_2Elist_2ECONS \\ & \quad A_27a)\ V0x)\ (ap\ (c_2Elist_2EVERSE\ A_27a)\ V1l)))))) \end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1l \in (ty_2Elist_2Elist \\ & \quad A_27a). ((p\ (ap\ V0P\ V1l)) \Rightarrow (\forall V2x \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\ & \quad c_2Elist_2ESNOC\ A_27a)\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & \quad A_27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in ((A_27b^{A_27a})^{A_27b}). (\forall V1e \in A_27b. (\forall V2x \in \\ & \quad A_27a. (\forall V3l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (ap\ (c_2Elist_2EFOLDL \\ & \quad A_27a\ A_27b)\ V0f)\ V1e)\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V2x)\ V3l)) = \\ & \quad (ap\ (ap\ V0f\ (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V0f)\ V1e)\ V3l)) \\ & \quad V2x)))))) \end{aligned} \tag{16}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in ((A_27a^{A_27b})^{A_27a}). (\forall V1e \in A_27a. (\forall V2l \in \\ & \quad (ty_2Elist_2Elist\ A_27b). ((ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27b \\ & \quad A_27a)\ V0f)\ V1e)\ V2l) = (ap\ (ap\ (ap\ (c_2Elist_2EFOLDR\ A_27b\ A_27a) \\ & \quad (\lambda V3x \in A_27b. (\lambda V4y \in A_27a. (ap\ (ap\ V0f\ V4y)\ V3x)))) V1e)\ (\\ & \quad ap\ (c_2Elist_2EVERSE\ A_27b)\ V2l)))))) \end{aligned}$$