

thm_2Erich_list_2EFOLDL_MAP2
(TMJY9mhR7tiMQmMg4ruMhaMQH3bfRp1v24n)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2Elist_2ECONS A.27a \in (((ty_2Elist_2Elist A.27a)^{(ty_2Elist_2Elist A.27a)})_{A.27a}) \quad (2)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Elist_2EMAP A.27a A.27b \in (((ty_2Elist_2Elist A.27b)^{(ty_2Elist_2Elist A.27b)})_{(A.27b)^{A.27a}}) \quad (3)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2Elist_2ENIL A.27a \in (ty_2Elist_2Elist A.27a) \quad (4)$$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap (ap (c_2Emin_2E_3D (2^{A.27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2Elist_2ESNOC A.27a \in (((ty_2Elist_2Elist A.27a)^{(ty_2Elist_2Elist A.27a)})_{A.27a}) \quad (5)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL \\ A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b^{A_27a})^{A_27b})}) \end{aligned} \quad (6)$$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ (\forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\ V0f)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27b))) \wedge (\forall V1f \in \\ (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\ A_27a).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS \\ A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ (ap\ V1f\ V2h)) \\ (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V3t))))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ (\forall V0f \in ((A_27b^{A_27a})^{A_27b}).(\forall V1e \in A_27b.((ap\ (\\ ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V0f)\ V1e)\ (c_2Elist_2ENIL \\ A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b^{A_27a})^{A_27b}).(\forall V3e \in \\ A_27b.(\forall V4x \in A_27a.(\forall V5l \in (ty_2Elist_2Elist\ A_27a). \\ ((ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c_2Elist_2ECONS \\ A_27a)\ V4x)\ V5l)) = (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V2f) \\ (ap\ (ap\ V2f\ V3e)\ V4x))\ V5l))))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A_27a}).(\forall V1x \in A_27a.(\forall V2l \in (\\ ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\ V0f)\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V1x)\ V2l)) = (ap\ (ap\ (c_2Elist_2ESNOC \\ A_27b)\ (ap\ V0f\ V1x))\ (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V0f)\ V2l)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A.27a)}), \\
& (((p\ (ap\ V0P\ (c.2Elist_2ENIL\ A.27a))) \wedge (\forall V1l \in (ty_2Elist_2Elist \\
& A.27a).(p\ (ap\ V0P\ V1l))) \Rightarrow (\forall V2x \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c.2Elist_2ESNOC\ A.27a\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A.27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in ((A.27b^{A.27a})^{A.27b}).(\forall V1e \in A.27b.(\forall V2x \in \\
& A.27a.(\forall V3l \in (ty_2Elist_2Elist\ A.27a).((ap\ (ap\ (ap\ (c.2Elist_2EFOLDL \\
& A.27a\ A.27b)\ V0f)\ V1e)\ (ap\ (ap\ (c.2Elist_2ESNOC\ A.27a)\ V2x)\ V3l)) = \\
& (ap\ (ap\ V0f\ (ap\ (ap\ (ap\ (c.2Elist_2EFOLDL\ A.27a\ A.27b)\ V0f)\ V1e)\ V3l)) \\
& V2x))))))
\end{aligned} \tag{14}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in ((A.27a^{A.27b})^{A.27a}).(\forall V1e \in \\
& A.27a.(\forall V2g \in (A.27b^{A.27c}).(\forall V3l \in (ty_2Elist_2Elist \\
& A.27c).((ap\ (ap\ (ap\ (c.2Elist_2EFOLDL\ A.27b\ A.27a)\ V0f)\ V1e)\ (ap \\
& (ap\ (c.2Elist_2EMAP\ A.27c\ A.27b)\ V2g)\ V3l)) = (ap\ (ap\ (ap\ (c.2Elist_2EFOLDL \\
& A.27c\ A.27a)\ (\lambda V4x \in A.27a.(\lambda V5y \in A.27c.(ap\ (ap\ V0f\ V4x)\ (\\
& ap\ V2g\ V5y))))))\ V1e)\ V3l))))))
\end{aligned}$$