

thm\_2Erich\_\_list\_2EFOLDR\_\_CONS\_\_NIL  
(TMH43eUxTcFTpJAyxCJUMzZPXFGMyyvtk7ip)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Elist\_2EFOLDR A\_27a A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27b})^{(A\_27b^{A\_27b})^{A\_27a}}) \quad (2)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (3)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (4)$$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A.27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (6)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & (\forall V0f \in ((A.27b^{A.27b})^{A.27a}).(\forall V1e \in A.27b.((ap\ ( \\ & ap\ (ap\ (c.2Elist.2EFOLDR\ A.27a\ A.27b)\ V0f)\ V1e)\ (c.2Elist.2ENIL \\ & A.27a)) = V1e))) \wedge (\forall V2f \in ((A.27b^{A.27b})^{A.27a}).(\forall V3e \in \\ & A.27b.(\forall V4x \in A.27a.(\forall V5l \in (ty.2Elist.2Elist\ A.27a). \\ & ((ap\ (ap\ (ap\ (c.2Elist.2EFOLDR\ A.27a\ A.27b)\ V2f)\ V3e)\ (ap\ (ap\ (c.2Elist.2ECONS \\ & A.27a)\ V4x)\ V5l)) = (ap\ (ap\ V2f\ V4x)\ (ap\ (ap\ (ap\ (c.2Elist.2EFOLDR \\ & A.27a\ A.27b)\ V2f)\ V3e)\ V5l)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty.2Elist.2Elist\ A.27a)}). \\ & (((p\ (ap\ V0P\ (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\ & A.27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ ( \\ & c.2Elist.2ECONS\ A.27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\ & A.27a).(p\ (ap\ V0P\ V3l)))) \end{aligned} \quad (10)$$

**Theorem 1**

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty.2Elist.2Elist\ A.27a).((ap\ (ap\ (ap\ (c.2Elist.2EFOLDR\ A.27a\ (ty.2Elist.2Elist\ A.27a))\ (c.2Elist.2ECONS\ A.27a))\ (c.2Elist.2ENIL\ A.27a))\ V0l) = V0l))$$