

thm_2Erich_list_2EFOLDR__FOLDL
(TMLw7dHS1BU7DG5iPMBpnVtGgiBCoMk4FPs)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})$

Definition 4 We define `c_2Ecombin_2ELEFT_ID` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27b^{A_27b})^{A_27a}).\lambda$

Definition 5 We define `c_2Ecombin_2ERIGHT_ID` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27b})^{A_27a}).\lambda$

Definition 6 We define `c_2Ecombin_2EASSOC` to be $\lambda A_27a : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21$

Definition 7 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 9 We define `c_2Ecombin_2EMONOID` to be $\lambda A_27a : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27a}).\lambda V1e \in A_27a.$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let `c_2Elist_2EFOLDR` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EFOLDR \\ A_27a A_27b \in (((A_27b^{(ty_2Elist_2Elist A_27a)})^{A_27b})^{(A_27b^{A_27b})^{A_27a}}) \end{aligned} \quad (2)$$

Let `c_2Elist_2ECONS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (3)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (4)$$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ESNOC\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (5)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL\ A_27a\ A_27b \in (((A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b)^{A_27a})^{A_27b}} \quad (6)$$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad (\forall V0f \in ((A_27b)^{A_27b})^{A_27a}). (\forall V1e \in A_27b. ((ap\ (\\ & \quad ap\ (ap\ (c_2Elist_2EFOLDR\ A_27a\ A_27b)\ V0f)\ V1e)\ (c_2Elist_2ENIL \\ & \quad A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b)^{A_27b})^{A_27a}). (\forall V3e \in \\ & \quad A_27b. (\forall V4x \in A_27a. (\forall V5l \in (ty_2Elist_2Elist\ A_27a). \\ & \quad ((ap\ (ap\ (ap\ (c_2Elist_2EFOLDR\ A_27a\ A_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & \quad A_27a)\ V4x)\ V5l)) = (ap\ (ap\ V2f\ V4x)\ (ap\ (ap\ (ap\ (c_2Elist_2EFOLDR \\ & \quad A_27a\ A_27b)\ V2f)\ V3e)\ V5l)))))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad (\forall V0f \in ((A_27b)^{A_27a})^{A_27b}). (\forall V1e \in A_27b. ((ap\ (\\ & \quad ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V0f)\ V1e)\ (c_2Elist_2ENIL \\ & \quad A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b)^{A_27a})^{A_27b}). (\forall V3e \in \\ & \quad A_27b. (\forall V4x \in A_27a. (\forall V5l \in (ty_2Elist_2Elist\ A_27a). \\ & \quad ((ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & \quad A_27a)\ V4x)\ V5l)) = (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V2f)\ \\ & \quad (ap\ (ap\ V2f\ V3e)\ V4x)\ V5l)))))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A.27a)}), \\
& (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A.27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& A.27a).(p\ (ap\ V0P\ V1t))) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ A.27a\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A.27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A.27a)}), \\
& (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A.27a))) \wedge (\forall V1l \in (ty_2Elist_2Elist \\
& A.27a).(p\ (ap\ V0P\ V1l))) \Rightarrow (\forall V2x \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c_2Elist_2ESNOC\ A.27a\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A.27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in ((A.27b^{A.27a})^{A.27b}).(\forall V1e \in A.27b.(\forall V2x \in \\
& A.27a.(\forall V3l \in (ty_2Elist_2Elist\ A.27a).((ap\ (ap\ (ap\ (c_2Elist_2EFOLDL \\
& A.27a\ A.27b)\ V0f)\ V1e)\ (ap\ (ap\ (c_2Elist_2ESNOC\ A.27a)\ V2x)\ V3l)) = \\
& (ap\ (ap\ V0f\ (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A.27a\ A.27b)\ V0f)\ V1e)\ V3l)) \\
& V2x))))))
\end{aligned} \tag{13}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in ((A.27a^{A.27a})^{A.27a}). \\
& (\forall V1e \in A.27a.((p\ (ap\ (ap\ (c_2Ecombin_2EMONOID\ A.27a)\ V0f) \\
& V1e)) \Rightarrow (\forall V2l \in (ty_2Elist_2Elist\ A.27a).((ap\ (ap\ (ap\ (c_2Elist_2EFOLDR \\
& A.27a\ A.27a)\ V0f)\ V1e)\ V2l) = (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A.27a \\
& A.27a)\ V0f)\ V1e)\ V2l))))))
\end{aligned}$$