

thm\_2Erich\_list\_2EIS\_PREFIX  
(TMSA7B3v4pzuJjbPG1px9PBNQfqwGHmB6NF)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (2)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (3)$$

Let  $c\_2Elist\_2EisPREFIX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EisPREFIX A\_27a \in ((2^{(ty\_2Elist\_2Elist A\_27a)})^{(ty\_2Elist\_2Elist A\_27a)}) \quad (4)$$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0l \in (\text{ty\_2Elist\_2Elist } \\ & A\_27a). (\forall V1h \in A\_27a. (\forall V2t \in (\text{ty\_2Elist\_2Elist } A\_27a). \\ & (\forall V3h1 \in A\_27a. (\forall V4t1 \in (\text{ty\_2Elist\_2Elist } A\_27a). \\ & (\forall V5h2 \in A\_27a. (\forall V6t2 \in (\text{ty\_2Elist\_2Elist } A\_27a). \\ & (((p (ap (ap (c\_2Elist\_2EisPREFIX A\_27a) (c\_2Elist\_2ENIL A\_27a)) \\ & V0l)) \Leftrightarrow \text{True}) \wedge (((p (ap (ap (c\_2Elist\_2EisPREFIX A\_27a) (ap (ap ( \\ & c\_2Elist\_2ECONS A\_27a) V1h) V2t)) (c\_2Elist\_2ENIL A\_27a))) \Leftrightarrow \text{False}) \wedge \\ & ((p (ap (ap (c\_2Elist\_2EisPREFIX A\_27a) (ap (ap (c\_2Elist\_2ECONS \\ & A\_27a) V3h1) V4t1)) (ap (ap (c\_2Elist\_2ECONS A\_27a) V5h2) V6t2))) \Leftrightarrow \\ & ((V3h1 = V5h2) \wedge (p (ap (ap (c\_2Elist\_2EisPREFIX A\_27a) V4t1) V6t2)))))))))))))) \end{aligned} \quad (6)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((\forall V0l \in (\text{ty\_2Elist\_2Elist } \\ & A\_27a). ((p (ap (ap (c\_2Elist\_2EisPREFIX A\_27a) (c\_2Elist\_2ENIL \\ & A\_27a)) V0l)) \Leftrightarrow \text{True})) \wedge ((\forall V1x \in A\_27a. (\forall V2l \in (\text{ty\_2Elist\_2Elist } \\ & A\_27a). ((p (ap (ap (c\_2Elist\_2EisPREFIX A\_27a) (ap (ap (c\_2Elist\_2ECONS \\ & A\_27a) V1x) V2l)) (c\_2Elist\_2ENIL A\_27a))) \Leftrightarrow \text{False}))) \wedge ((\forall V3x1 \in \\ & A\_27a. (\forall V4l1 \in (\text{ty\_2Elist\_2Elist } A\_27a). (\forall V5x2 \in \\ & A\_27a. (\forall V6l2 \in (\text{ty\_2Elist\_2Elist } A\_27a). ((p (ap (ap (c\_2Elist\_2EisPREFIX \\ & A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V5x2) V6l2)) (ap (ap (c\_2Elist\_2ECONS \\ & A\_27a) V3x1) V4l1))) \Leftrightarrow ((V3x1 = V5x2) \wedge (p (ap (ap (c\_2Elist\_2EisPREFIX \\ & A\_27a) V6l2) V4l1)))))))))))))) \end{aligned}$$