thm_2Erich__list_2EIS__PREFIX__APPEND (TMT8gmQ3zY1p8ScN28Ldgd1A74fVeuhLcGP)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{2}$$

Definition 4 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 5 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 7 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{3}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{4}$$

Let $c\_2Elist\_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EisPREFIX\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27} \tag{5}$$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{7}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{10}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))\ V0l) = V0l)) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3h \in A\_27a.((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2)))))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a0 \in A\_27a.(\forall V1a1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2a0\_27 \in A\_27a.(\forall V3a1\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0a0)\ V1a1) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0\_27)\ V3a1\_27)) \Leftrightarrow ((V0a0 = V2a0\_27) \wedge (V1a1 = V3a1\_27))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1a0 \in A\_27a.(\neg((ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a0)\ V0a1) = (c\_2Elist\_2ENIL\ A\_27a))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))\ V0l)) \Leftrightarrow True)) \wedge ((\forall V1x \in A\_27a.(\forall V2l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1x)\ V2l))\ (c\_2Elist\_2ENIL\ A\_27a))) \Leftrightarrow False))) \wedge (\forall V3x1 \in A\_27a.(\forall V4l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V5x2 \in A\_27a.(\forall V6l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5x2)\ V6l2))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3x1)\ V4l1))) \Leftrightarrow ((V3x1 = V5x2) \wedge (p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ V6l2)\ V4l1)))))))))) \tag{18}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ V1l2)\ V0l1)) \Leftrightarrow (\exists V2l \in (ty\_2Elist\_2Elist\ A\_27a).(V0l1 = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l2)\ V2l))))))$$