

thm\_2Erich\_\_list\_2EIS\_\_PREFIX\_\_APPEND3  
(TMVFxtNosCCz1gUCLtRLeZ7us2LAA36LMyM)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))A\_27a) \quad (2)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (3)$$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EAPPEND A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))(ty\_2Elist\_2Elist A\_27a)) \quad (4)$$

Let  $c\_2Elist\_2EisPREFIX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EisPREFIX\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (5)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p\ V0t)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(ap\ (ap\ (c\_2Elist\_2EAPPEND \\ & A\_27a)\ V0l)\ (c\_2Elist\_2ENIL\ A\_27a)) = V0l) \wedge (\forall V1l \in (ty\_2Elist\_2Elist \\ & A\_27b).(ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27b)\ (c\_2Elist\_2ENIL\ A\_27b)) \\ & V1l) = V1l)) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist \\ & A\_27a).(p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ (c\_2Elist\_2ENIL \\ & A\_27a))\ V0l)) \Leftrightarrow True) \wedge ((\forall V1x \in A\_27a.(\forall V2l \in (ty\_2Elist\_2Elist \\ & A\_27a).(p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\ & A\_27a)\ V1x)\ V2l))\ (c\_2Elist\_2ENIL\ A\_27a))) \Leftrightarrow False)) \wedge (\forall V3x1 \in \\ & A\_27a.(\forall V4l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V5x2 \in \\ & A\_27a.(\forall V6l2 \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX \\ & A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5x2)\ V6l2))\ (ap\ (ap\ (c\_2Elist\_2ECONS \\ & A\_27a)\ V3x1)\ V4l1))) \Leftrightarrow ((V3x1 = V5x2) \wedge (p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX \\ & A\_27a)\ V6l2)\ V4l1))))))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Elist\_2Elist \\ & A\_27a).(\forall V1b \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2c \in \\ & (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a) \\ & (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0a)\ V1b))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND \\ & A\_27a)\ V0a)\ V2c))) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ V1b) \\ & V2c)))))) \end{aligned} \quad (9)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Elist\_2Elist \\ & A\_27a).(\forall V1a \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX \\ & A\_27a)\ V1a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1a)\ V0c)))))) \end{aligned}$$