

thm_2Erich__list_2EIS__PREFIX__LENGTH__ANTI (TMMc1DAL1WBkUnoiECdav3Bj3x2Uqt2C9sj)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$ Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 4 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{4}$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \tag{5}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{6}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{7}$$

Definition 5 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 7 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (8)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (9)$$

Let $c_2Elist_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EisPREFIX A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (10)$$

Definition 8 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 11 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg (p V0t)))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge (p \ V1t2) \wedge (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (21)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (((ap (c.2Elist.2ELENGTH A.27a) (c.2Elist.2ENIL A.27a)) = c.2Enum.2E0) \wedge (\forall V0h \in A.27a.(\forall V1t \in (ty.2Elist.2Elist A.27a).((ap (c.2Elist.2ELENGTH A.27a) (ap (ap (c.2Elist.2ECONS A.27a) V0h) V1t)) = (ap c.2Enum.2ESUC (ap (c.2Elist.2ELENGTH A.27a) V1t)))))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{(ty.2Elist.2Elist A.27a)}).(((p (ap V0P (c.2Elist.2ENIL A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist A.27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A.27a.(p (ap V0P (ap (ap (c.2Elist.2ECONS A.27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist A.27a).(p (ap V0P V3l)))))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0l \in (ty.2Elist.2Elist A.27a).((V0l = (c.2Elist.2ENIL A.27a)) \vee (\exists V1h \in A.27a.(\exists V2t \in (ty.2Elist.2Elist A.27a).(V0l = (ap (ap (c.2Elist.2ECONS A.27a) V1h) V2t)))))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0a0 \in A.27a.(\forall V1a1 \in (ty.2Elist.2Elist A.27a).(\forall V2a0.27 \in A.27a.(\forall V3a1.27 \in (ty.2Elist.2Elist A.27a).(((ap (ap (c.2Elist.2ECONS A.27a) V0a0) V1a1) = (ap (ap (c.2Elist.2ECONS A.27a) V2a0.27) V3a1.27)) \Leftrightarrow ((V0a0 = V2a0.27) \wedge (V1a1 = V3a1.27)))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a1 \in (ty_2Elist_2Elist\ A_27a).(\forall V1a0 \in A_27a.(\neg((ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1a0)\ V0a1) = (c_2Elist_2ENIL\ A_27a)))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist\ A_27a).(((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l) = c_2Enum_2E0) \Leftrightarrow (V0l = (c_2Elist_2ENIL\ A_27a)))) \quad (35)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap\ c_2Enum_2ESUC\ V0m) = (ap\ c_2Enum_2ESUC\ V1n)) \Rightarrow (V0m = V1n))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ (c_2Elist_2ENIL\ A_27a))\ V0l) \Leftrightarrow True)) \wedge ((\forall V1x \in A_27a.(\forall V2l \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1x)\ V2l))\ (c_2Elist_2ENIL\ A_27a)) \Leftrightarrow False))) \wedge (\forall V3x1 \in A_27a.(\forall V4l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V5x2 \in A_27a.(\forall V6l2 \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V5x2)\ V6l2))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3x1)\ V4l1)) \Leftrightarrow ((V3x1 = V5x2) \wedge (p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ V6l2)\ V4l1)))))))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Elist_2Elist\ A_27a).(p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ V0x)\ V0x))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (53)$$

Theorem 1

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V_0x \in (\text{ty_2Elist_2Elist } A_{27a}). (\forall V_1y \in (\text{ty_2Elist_2Elist } A_{27a}). (((p \text{ (ap (ap (c_2Elist_2EisPREFIX } A_{27a}) V_0x) V_1y)) \wedge ((\text{ap (c_2Elist_2ELENGTH } A_{27a}) V_0x) = (\text{ap (c_2Elist_2ELENGTH } A_{27a}) V_1y))) \Leftrightarrow (V_0x = V_1y))))))$$