

thm\_2Erich\_list\_2EIS\_PREFIX\_PREFIX  
(TMZRbhE72E1UVZoz4z2VydU97iVkfUUY62)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2EisPREFIX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EisPREFIX A\_27a \in ((2^{(ty\_2Elist\_2Elist A\_27a)})^{(ty\_2Elist\_2Elist A\_27a)}) \quad (2)$$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{A\_27a})^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a} \quad (3)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (4)$$

**Definition 11** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$ .

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (5)$$

Let  $c\_2Erich\_list\_2ESPLITP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Erich\_list\_2ESPLITP\ A\_27a \in (((ty\_2Epair\_2Eprod\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27a))^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \quad (6)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (7)$$

**Definition 12** We define  $c\_2Erich\_list\_2EPREFIX$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1l \in (ty\_2Elist\_2Elist\ A\_27a)$ .

Assume the following.

$$True \quad (8)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (12)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ &(p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ &(((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (13) \end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}), \\ & (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\ & A\_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a. (p\ (ap\ V0P\ (ap\ (ap\ ( \\ & c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ & A\_27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist \\ & A\_27a). ((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ (c\_2Elist\_2ENIL \\ & A\_27a)\ V0l)) \Leftrightarrow True)) \wedge ((\forall V1x \in A\_27a. (\forall V2l \in (ty\_2Elist\_2Elist \\ & A\_27a). ((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\ & A\_27a)\ V1x)\ V2l))\ (c\_2Elist\_2ENIL\ A\_27a))) \Leftrightarrow False))) \wedge (\forall V3x1 \in \\ & A\_27a. (\forall V4l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V5x2 \in \\ & A\_27a. (\forall V6l2 \in (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX \\ & A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5x2)\ V6l2))\ (ap\ (ap\ (c\_2Elist\_2ECONS \\ & A\_27a)\ V3x1)\ V4l1))) \Leftrightarrow ((V3x1 = V5x2) \wedge (p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX \\ & A\_27a)\ V6l2)\ V4l1)))))))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& (\forall V0P \in (2^{A\_27a}).((ap\ (ap\ (c\_2Erich\_list\_2EPREFIX\ A\_27a) \\
V0P)\ (c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1P \in \\
& (2^{A\_27b}).(\forall V2x \in A\_27b.(\forall V3l \in (ty\_2Elist\_2Elist \\
& A\_27b).((ap\ (ap\ (c\_2Erich\_list\_2EPREFIX\ A\_27b)\ V1P)\ (ap\ (ap\ ( \\
c\_2Elist\_2ECONS\ A\_27b)\ V2x)\ V3l)) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND \\
& (ty\_2Elist\_2Elist\ A\_27b))\ (ap\ V1P\ V2x))\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& A\_27b)\ V2x)\ (ap\ (ap\ (c\_2Erich\_list\_2EPREFIX\ A\_27b)\ V1P)\ V3l))) \\
& (c\_2Elist\_2ENIL\ A\_27b))))))
\end{aligned} \tag{19}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1l \in \\
& (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a) \\
& (ap\ (ap\ (c\_2Erich\_list\_2EPREFIX\ A\_27a)\ V0P)\ V1l))\ V1l))))
\end{aligned}$$