thm_2Erich_list_2EIS_PREFIX_TRANS (TMMv8m4UK5w78tAAipWG8Use99CoH9LEWDW)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A$.**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \qquad (1)$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$
$$\qquad (2)$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist$$
$$A\_27a)$$
$$\qquad (3)$$

Let $c\_2Elist\_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EisPREFIX\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27}$$
$$\qquad (4)$$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 8 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\textsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\textsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \textsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \textsf{c\_2Ebool\_2E}$

Assume the following.
$$True \tag{5}$$

Assume the following.
$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{6}$$

Assume the following.
$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in\\ A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{7}$$

Assume the following.
$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \tag{8}$$

Assume the following.
$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{9}$$

Assume the following.
$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow\\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge\\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{10}$$

Assume the following.
$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow\\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))))))) \tag{11}$$

Assume the following.
$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge\\ ((\neg False) \Leftrightarrow True))) \tag{12}$$

Assume the following.
$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow\\ True)) \tag{13}$$

Assume the following.
$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in\\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{14}$$

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Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))))) \tag{15}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \tag{16}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A))))) \tag{17}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{18}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B))))) \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{20}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).(\forall V1v \in A\_27a.((\forall V2x \in A\_27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (ap\ V0f\ V1v))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).\\ (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist$$
$$A\_27a).((V0l = (c\_2Elist\_2ENIL\ A\_27a)) \vee (\exists V1h \in A\_27a.($$
$$\exists V2t \in (ty\_2Elist\_2Elist\ A\_27a).(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS \tag{24}$$
$$A\_27a)\ V1h)\ V2t))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist$$
$$A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ (c\_2Elist\_2ENIL$$
$$A\_27a))\ V0l)) \Leftrightarrow True)) \wedge ((\forall V1x \in A\_27a.(\forall V2l \in (ty\_2Elist\_2Elist$$
$$A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS$$
$$A\_27a)\ V1x)\ V2l))\ (c\_2Elist\_2ENIL\ A\_27a))) \Leftrightarrow False))) \wedge (\forall V3x1 \in$$
$$A\_27a.(\forall V4l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V5x2 \in$$
$$A\_27a.(\forall V6l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX$$
$$A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5x2)\ V6l2))\ (ap\ (ap\ (c\_2Elist\_2ECONS$$
$$A\_27a)\ V3x1)\ V4l1))) \Leftrightarrow ((V3x1 = V5x2) \wedge (p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX$$
$$A\_27a)\ V6l2)\ V4l1)))))))))))$$
$$\tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Elist\_2Elist$$
$$A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ (c\_2Elist\_2ENIL$$
$$A\_27a))\ V0x)) \wedge ((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ V0x)\ (c\_2Elist\_2ENIL$$
$$A\_27a))) \Leftrightarrow (V0x = (c\_2Elist\_2ENIL\ A\_27a)))))$$
$$\tag{26}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{27}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow$$
$$(((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow$$
$$((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{30}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{31}$$

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Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ($$
$$(p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \lor ((p \ V1q) \lor (p \ V2r))) \land (((p \ V0p) \lor ((\neg ($$
$$p \ V2r)) \lor (\neg (p \ V1q)))) \land (((p \ V1q) \lor ((\neg (p \ V2r)) \lor (\neg (p \ V0p)))) \land ((p \ V2r) \lor$$
$$((\neg (p \ V1q)) \lor (\neg (p \ V0p))))))))))))$$

$$(32)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ($$
$$(p \ V1q) \land (p \ V2r))) \Leftrightarrow (((p \ V0p) \lor ((\neg (p \ V1q)) \lor (\neg (p \ V2r)))) \land (((p \ V1q) \lor$$
$$(\neg (p \ V0p))) \land ((p \ V2r) \lor (\neg (p \ V0p))))))))))$$

$$(33)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ($$
$$(p \ V1q) \lor (p \ V2r))) \Leftrightarrow (((p \ V0p) \lor (\neg (p \ V1q))) \land (((p \ V0p) \lor (\neg (p \ V2r))) \land$$
$$((p \ V1q) \lor ((p \ V2r) \lor (\neg (p \ V0p)))))))))))$$

$$(34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ($$
$$(p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \lor (p \ V1q)) \land (((p \ V0p) \lor (\neg (p \ V2r))) \land (($$
$$\neg (p \ V1q)) \lor ((p \ V2r) \lor (\neg (p \ V0p)))))))))))$$

$$(35)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p) \Leftrightarrow (\neg (p \ V1q))) \Leftrightarrow (((p \ V0p) \lor$$
$$(p \ V1q)) \land ((\neg (p \ V1q)) \lor (\neg (p \ V0p)))))))$$

$$(36)$$

Theorem 1

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in (ty\_2Elist\_2Elist$$
$$A\_27a).(\forall V1y \in (ty\_2Elist\_2Elist \ A\_27a).(\forall V2z \in$$
$$(ty\_2Elist\_2Elist \ A\_27a).(((p \ (ap \ (ap \ (c\_2Elist\_2EisPREFIX \ A\_27a)$$
$$V1y) \ V0x)) \land (p \ (ap \ (ap \ (c\_2Elist\_2EisPREFIX \ A\_27a) \ V2z) \ V1y))) \Rightarrow$$
$$(p \ (ap \ (ap \ (c\_2Elist\_2EisPREFIX \ A\_27a) \ V2z) \ V0x))))))$$