

thm\_2Erich\_\_list\_2EIS\_\_SUFFIX\_\_REFL  
(TMabshRJgnsEX2h4goo5mpZk3y66mMqYq6s)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))A\_27a) \quad (2)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (3)$$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EAPPEND A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))(ty\_2Elist\_2Elist A\_27a)) \quad (4)$$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$  **if**  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 9** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) P)))$

Let  $c\_2Erich\_list\_2EIS\_SUFFIX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Erich\_list\_2EIS\_SUFFIX A\_27a \in ((2^{(ty\_2Elist\_2Elist A\_27a)})(ty\_2Elist\_2Elist A\_27a)) \quad (5)$$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t))))) \quad (8)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0a \in A\_27a.(\exists V1x \in A\_27a.(V1x = V0a))) \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & ((\forall V0l \in (ty\_2Elist\_2Elist A\_27a).((ap (ap (c\_2Elist\_2EAPPEND A\_27a) (c\_2Elist\_2ENIL A\_27a)) V0l) = V0l) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist A\_27a).(\forall V3h \in A\_27a.((ap (ap (c\_2Elist\_2EAPPEND A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V3h) V1l1)) V2l2) = (ap (ap (c\_2Elist\_2ECONS A\_27a) V3h) (ap (ap (c\_2Elist\_2EAPPEND A\_27a) V1l1) V2l2))))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & ((\forall V0l1 \in (ty\_2Elist\_2Elist A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist A\_27a).(\forall V2l3 \in (ty\_2Elist\_2Elist A\_27a).(((ap (ap (c\_2Elist\_2EAPPEND A\_27a) V0l1) V1l2) = (ap (ap (c\_2Elist\_2EAPPEND A\_27a) V0l1) V2l3)) \Leftrightarrow (V1l2 = V2l3)))))) \wedge (\forall V3l1 \in (ty\_2Elist\_2Elist A\_27a).(\forall V4l2 \in (ty\_2Elist\_2Elist A\_27a).(\forall V5l3 \in (ty\_2Elist\_2Elist A\_27a).(((ap (ap (c\_2Elist\_2EAPPEND A\_27a) V4l2) V3l1) = (ap (ap (c\_2Elist\_2EAPPEND A\_27a) V5l3) V3l1)) \Leftrightarrow (V4l2 = V5l3)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist \\ & A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Erich\_list\_2EIS\_SUFFIX \\ & A\_27a)\ V0l1)\ V1l2)) \Leftrightarrow (\exists V2l \in (ty\_2Elist\_2Elist\ A\_27a). \\ & V0l1 = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V2l)\ V1l2)))))) \\ & \hspace{15em} (12) \end{aligned}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (c\_2Erich\_list\_2EIS\_SUFFIX\ A\_27a)\ V0l)\ V0l)))$$