

thm_2Erich_list_2ELASTN
 (TMd1iMo183ABMwXyVBSWpPvw2zQiqqWYWt8)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (2)$$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ESNOC A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (3)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (4)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (5)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (8)$$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (9)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Definition 6 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ETAKE\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EREVERSE\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \quad (12)$$

Definition 9 We define $c_2Erich_list_2ELASTN$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.\lambda V1xs \in (ty_2Enum_2Enum$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (((ap\ (c_2Elist_2EREVERSE\ A_27a) \\ & (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27a)) \wedge (\forall V0h \in \\ & A_27a. (\forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2EREVERSE \\ A_27a) (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Elist_2EAPPEND \\ A_27a) (ap\ (c_2Elist_2EREVERSE\ A_27a)\ V1t)) (ap\ (ap\ (c_2Elist_2ECONS \\ A_27a)\ V0h)\ (c_2Elist_2ENIL\ A_27a)))))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0x \in A_27a. (\forall V1l \in \\ & (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V0x) \\ V1l) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V1l) (ap\ (ap\ (c_2Elist_2ECONS \\ A_27a)\ V0x) (c_2Elist_2ENIL\ A_27a)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0x \in A_27a. (\forall V1l \in \\ & (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2EREVERSE\ A_27a) (ap \\ (ap\ (c_2Elist_2ESNOC\ A_27a)\ V0x)\ V1l)) = (ap\ (ap\ (c_2Elist_2ECONS \\ A_27a)\ V0x) (ap\ (c_2Elist_2EREVERSE\ A_27a)\ V1l)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0l \in (ty_2Elist_2Elist \\ A_27a). ((ap\ (ap\ (c_2Elist_2ETAKE\ A_27a)\ c_2Enum_2E0)\ V0l) = (c_2Elist_2ENIL \\ A_27a))) \wedge (\forall V1n \in ty_2Enum_2Enum. (\forall V2x \in A_27a. (\\ \forall V3l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2ETAKE \\ A_27a) (ap\ c_2Enum_2ESUC\ V1n)) (ap\ (ap\ (c_2Elist_2ECONS\ A_27a) \\ V2x)\ V3l)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2x) (ap\ (ap\ (c_2Elist_2ETAKE \\ A_27a)\ V1n)\ V3l)))))))) \end{aligned} \quad (20)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Erich_list_2ELASTN \\ A_27a)\ c_2Enum_2E0)\ V0l) = (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1n \in \\ & ty_2Enum_2Enum. (\forall V2x \in A_27b. (\forall V3l \in (ty_2Elist_2Elist \\ A_27b). ((ap\ (ap\ (c_2Erich_list_2ELASTN\ A_27b) (ap\ c_2Enum_2ESUC \\ V1n)) (ap\ (ap\ (c_2Elist_2ESNOC\ A_27b)\ V2x)\ V3l)) = (ap\ (ap\ (c_2Elist_2ESNOC \\ A_27b)\ V2x) (ap\ (ap\ (c_2Erich_list_2ELASTN\ A_27b)\ V1n)\ V3l)))))))) \end{aligned}$$