

thm\_2Erich\_\_list\_2ELASTN\_\_APPEND1  
(TMV8HSADGSVjMVgr6yVDPNWKZeH9nT9Kir3)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_F$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (c\_2Enum\_2ESUC\_REP m))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 11** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 13** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (6)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (7)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (8)$$

Let  $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ESNOC\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (9)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (10)$$

**Definition 14** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (11)$$

Let  $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETAKE\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 15** We define  $c\_2Erich\_list\_2ELASTN$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum. \lambda V1xs \in (ty$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (13)$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\ & (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ (ap\ c\_2Enum\_2ESUC\ V0n))\ (ap \\ & c\_2Enum\_2ESUC\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0n) \\ & V1m)))))) \end{aligned} \quad (15)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ (ap\ c\_2Enum\_2ESUC\ V0n))\ c\_2Enum\_2E0)))) \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (((ap\ (ap\ c\_2Earithmetic\_2E\_2D \\ & c\_2Enum\_2E0)\ V0m) = c\_2Enum\_2E0) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2D \\ & V0m)\ c\_2Enum\_2E0) = V0m))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\ & (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ c\_2Enum\_2ESUC\ V0n))\ (ap\ c\_2Enum\_2ESUC \\ & V1m)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ V1m)))) \end{aligned} \quad (18)$$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (22)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a) \\
& (c\_2Elist\_2ENIL\ A\_27a)) = c\_2Enum\_2E0) \wedge (\forall V0h \in A\_27a.( \\
& \forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).(ap\ (c\_2Elist\_2ELENGTH \\
& A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ c\_2Enum\_2ESUC \\
& (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1l \in \\
& (ty\_2Elist\_2Elist\ A\_27a).(ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap \\
& (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V0x)\ V1l)) = (ap\ c\_2Enum\_2ESUC\ (ap\ ( \\
& c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist \\
& A\_27a).(\forall V1x \in A\_27a.(\forall V2l2 \in (ty\_2Elist\_2Elist \\
& A\_27a).(ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ (ap\ (ap\ (c\_2Elist\_2ESNOC \\
& A\_27a)\ V1x)\ V2l2)) = (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V1x)\ (ap\ (ap \\
& (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V2l2))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). \\
& (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1l \in (ty\_2Elist\_2Elist \\
& A\_27a).(p\ (ap\ V0P\ V1l)) \Rightarrow (\forall V2x \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ ( \\
& c\_2Elist\_2ESNOC\ A\_27a)\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\
& A\_27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ c\_2Enum\_2E0)) \wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c\_2Enum\_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Erich\_list\_2ELASTN \\
& A\_27a)\ c\_2Enum\_2E0)\ V0l) = (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1n \in \\
& ty\_2Enum\_2Enum.(\forall V2x \in A\_27b.(\forall V3l \in (ty\_2Elist\_2Elist \\
& A\_27b).((ap\ (ap\ (c\_2Erich\_list\_2ELASTN\ A\_27b)\ (ap\ c\_2Enum\_2ESUC \\
& V1n))\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27b)\ V2x)\ V3l)) = (ap\ (ap\ (c\_2Elist\_2ESNOC \\
& A\_27b)\ V2x)\ (ap\ (ap\ (c\_2Erich\_list\_2ELASTN\ A\_27b)\ V1n)\ V3l)))))) \\
& \hspace{15em} (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND \\
& A\_27a)\ V0l)\ (c\_2Elist\_2ENIL\ A\_27a)) = V0l)) \wedge (\forall V1l \in (ty\_2Elist\_2Elist \\
& A\_27b).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27b)\ (c\_2Elist\_2ENIL\ A\_27b)) \\
& V1l) = V1l))) \\
& \hspace{15em} (31)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l2 \in (ty\_2Elist\_2Elist \\
& A\_27a).(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\
& (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l2))\ V1n)) \Rightarrow (\forall V2l1 \in (ty\_2Elist\_2Elist \\
& A\_27a).((ap\ (ap\ (c\_2Erich\_list\_2ELASTN\ A\_27a)\ V1n)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND \\
& A\_27a)\ V2l1)\ V0l2)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ ( \\
& c\_2Erich\_list\_2ELASTN\ A\_27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1n) \\
& (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l2))))\ V2l1))\ V0l2))))))
\end{aligned}$$