thm_2Erich_list_2ELASTN__LASTN (TMSvYR-WshG6urpRTN4MrN7W6WJ9b9FsA688)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty\_2Enum\_2Enum \tag{1}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o \ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap \ (ap \ (\text{c\_2Emin\_2E\_3D} \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \ (ap \ (\text{c\_2Emin\_2E\_3D} \ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap \ (\text{c\_2Ebool\_2E\_21} \ 2) \ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o \ (p \ P \Rightarrow p \ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap \ (ap \ \text{c\_2Emin\_2E\_3D\_3D\_3E} \ V0t) \ \text{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap \ (\text{c\_2Ebool\_2E\_21} \ 2) \ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap \ c\_2Enum\_2EABS\_\_num \ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if} \ (\exists x \in A.p \ (ap \ P \ x)) \ \textbf{then} \ (the \ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\textsf{c\_2Emin\_2E\_40}$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\textsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{5}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$
$$\tag{6}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$\tag{7}$$

Let $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ESNOC\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$
$$\tag{8}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist$$
$$A\_27a)$$
$$\tag{9}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{10}$$

Definition 14 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$\tag{11}$$

Let $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETAKE\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum})$$
$$\tag{12}$$

Definition 15 *We define* c_2Erich__list_2ELASTN *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1xs \in (ty$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))\ (ap$$
$$\mathsf{c\_2Enum\_2ESUC}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)$$
$$V1m))))) \tag{13}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\mathsf{c\_2Enum\_2E0})\ V0n))) \tag{14}$$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p$$
$$V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True\lor(p\ V0t))\Leftrightarrow True)\land((((p\ V0t)\lor True)\Leftrightarrow True)\land$$
$$(((False\lor(p\ V0t))\Leftrightarrow(p\ V0t))\land((((p\ V0t)\lor False)\Leftrightarrow(p\ V0t))\land(((p\ V0t)\lor$$
$$(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\land((((p\ V0t)\Rightarrow True)\Leftrightarrow$$
$$True)\land(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\land((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\land(($$
$$(p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t))))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow$$
$$True)) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\land((((p\ V0t)\Leftrightarrow True)\Leftrightarrow$$
$$(p\ V0t))\land(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\land(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg($$
$$p\ V0t))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)$$
$$(c\_2Elist\_2ENIL\ A\_27a)) = \mathsf{c\_2Enum\_2E0})\land(\forall V0h \in A\_27a.($$
$$\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH$$
$$A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$(ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t)))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1l \in$$
$$(ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap$$
$$(ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V0x)\ V1l)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ ($$
$$c\_2Elist\_2ELENGTH\ A\_27a)\ V1l)))))$$

$$(23)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).$$
$$((((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1l \in (ty\_2Elist\_2Elist$$
$$A\_27a).((p\ (ap\ V0P\ V1l)) \Rightarrow (\forall V2x \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ ($$
$$c\_2Elist\_2ESNOC\ A\_27a)\ V2x)\ V1l))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist$$
$$A\_27a).(p\ (ap\ V0P\ V3l))))))$$

$$(24)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = \mathsf{c\_2Enum\_2E0})))$$

$$(25)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})) \wedge$$
$$(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n)))))$$

$$(26)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_rec\_2E\_3C}$$
$$V0n)\ \mathsf{c\_2Enum\_2E0}))))$$

$$(27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$(\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (\mathsf{c\_2Erich\_list\_2ELASTN}$$
$$A\_27a)\ \mathsf{c\_2Enum\_2E0})\ V0l) = (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1n \in$$
$$ty\_2Enum\_2Enum.(\forall V2x \in A\_27b.(\forall V3l \in (ty\_2Elist\_2Elist$$
$$A\_27b).((ap\ (ap\ (\mathsf{c\_2Erich\_list\_2ELASTN}\ A\_27b)\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n))\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27b)\ V2x)\ V3l)) = (ap\ (ap\ (c\_2Elist\_2ESNOC$$
$$A\_27b)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Erich\_list\_2ELASTN}\ A\_27b)\ V1n)\ V3l)))))))$$

$$(28)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist$$
$$A\_27a).(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2m \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V2m)\ (ap\ (c\_2Elist\_2ELENGTH$$
$$A\_27a)\ V0l))) \Rightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2m)) \Rightarrow$$
$$((ap\ (ap\ (\mathsf{c\_2Erich\_list\_2ELASTN}\ A\_27a)\ V1n)\ (ap\ (ap\ (\mathsf{c\_2Erich\_list\_2ELASTN}$$
$$A\_27a)\ V2m)\ V0l)) = (ap\ (ap\ (\mathsf{c\_2Erich\_list\_2ELASTN}\ A\_27a)\ V1n)$$
$$V0l)))))))$$