

thm\_2Erich\_\_list\_2ELASTN\_\_LENGTH\_\_ID  
(TMdgJxAGkPwnSkbda5KeVPjBv9uriNBoyUe)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Elist\_2ECONS A.27a \in (((ty\_2Elist\_2Elist A.27a)^{(ty\_2Elist\_2Elist A.27a)})^{A.27a}) \quad (2)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (3)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Elist\_2ELENGTH A.27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist A.27a)}) \quad (4)$$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

Let  $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Elist\_2ESNOC A.27a \in (((ty\_2Elist\_2Elist A.27a)^{(ty\_2Elist\_2Elist A.27a)})^{A.27a}) \quad (5)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (6)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{omega}) \quad (7)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (8)$$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 5** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num ($

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (9)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (10)$$

**Definition 6** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2EREVERSE A\_27a \in ((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)}) \quad (11)$$

Let  $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ETAKE A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 7** We define  $c\_2Erich\_list\_2ELASTN$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum. \lambda V1xs \in (ty\_2Enum\_2Enum)^n$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. (ap (c\_2Ebool\_2E\_21 2) V2t))))$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (((ap (c\_2Elist\_2ELENGTH A\_27a) (c\_2Elist\_2ENIL A\_27a)) = c\_2Enum\_2E0) \wedge (\forall V0h \in A\_27a. (\forall V1t \in (ty\_2Elist\_2Elist A\_27a). ((ap (c\_2Elist\_2ELENGTH A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V0h) V1t)) = (ap c\_2Enum\_2ESUC (ap (c\_2Elist\_2ELENGTH A\_27a) V1t))))))) \quad (15)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1l \in \\ (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap \\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V0x)\ V1l)) = (ap\ c\_2Enum\_2ESUC\ (ap\ ( \\ c\_2Elist\_2ELENGTH\ A\_27a)\ V1l)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). \\ (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1l \in (ty\_2Elist\_2Elist \\ A\_27a). ((p\ (ap\ V0P\ V1l)) \Rightarrow (\forall V2x \in A\_27a. (p\ (ap\ V0P\ (ap\ (ap\ ( \\ c\_2Elist\_2ESNOC\ A\_27a)\ V2x)\ V1l))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ A\_27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (ap\ (c\_2Erich\_list\_2ELASTN \\ A\_27a)\ c\_2Enum\_2E0)\ V0l) = (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1n \in \\ ty\_2Enum\_2Enum. (\forall V2x \in A\_27b. (\forall V3l \in (ty\_2Elist\_2Elist \\ A\_27b). ((ap\ (ap\ (c\_2Erich\_list\_2ELASTN\ A\_27b)\ (ap\ c\_2Enum\_2ESUC \\ V1n))\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27b)\ V2x)\ V3l)) = (ap\ (ap\ (c\_2Elist\_2ESNOC \\ A\_27b)\ V2x)\ (ap\ (ap\ (c\_2Erich\_list\_2ELASTN\ A\_27b)\ V1n)\ V3l))))))) \end{aligned} \quad (18)$$

### Theorem 1

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ A\_27a). ((ap\ (ap\ (c\_2Erich\_list\_2ELASTN\ A\_27a)\ (ap\ (c\_2Elist\_2ELENGTH \\ A\_27a)\ V0l))\ V0l) = V0l)) \end{aligned}$$