

thm_2Erich__list_2ELENGTH__BUTLASTN
(TMV48cxS4mzjHn4sbfz79BxdxHnQNoocjxt)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (3)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (4)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (c_2Enum_2ESUC_REP m))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (5)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (6)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (7)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ELENGTH A_27a \in (ty_2Enum_2Enum)^{(ty_2Elist_2Elist A_27a)} \quad (8)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (9)$$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ESNOC A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (10)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (11)$$

Definition 14 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2EREVERSE A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)}) \quad (12)$$

Let $c_2Elist_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2EDROP A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 15 We define $c_Erich_list_2EBUTLASTN$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.\lambda V1xs$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(\\ & (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0n)) (ap \\ & c_2Enum_2ESUC V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0n) \\ & V1m)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2D \\ & c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D \\ & V0m) c_2Enum_2E0) = V0m))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(\\ & (ap (ap c_2Earithmetic_2E_2D (ap c_2Enum_2ESUC V0n)) (ap c_2Enum_2ESUC \\ & V1m)) = (ap (ap c_2Earithmetic_2E_2D V0n) V1m)))) \end{aligned} \quad (16)$$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. ((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (((ap\ (c.2Elist.2ELENGTH\ A.27a) \\ & (c.2Elist.2ENIL\ A.27a)) = c.2Enum.2E0) \wedge (\forall V0h \in A.27a. (\\ & \forall V1t \in (ty.2Elist.2Elist\ A.27a). ((ap\ (c.2Elist.2ELENGTH \\ A.27a)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a)\ V0h)\ V1t)) = (ap\ c.2Enum.2ESUC \\ & (ap\ (c.2Elist.2ELENGTH\ A.27a)\ V1t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0x \in A.27a. (\forall V1l \in \\ & (ty.2Elist.2Elist\ A.27a). ((ap\ (c.2Elist.2ELENGTH\ A.27a)\ (ap \\ & (ap\ (c.2Elist.2ESNOC\ A.27a)\ V0x)\ V1l)) = (ap\ c.2Enum.2ESUC\ (ap\ (\\ & c.2Elist.2ELENGTH\ A.27a)\ V1l)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0P \in (2^{(ty.2Elist.2Elist\ A.27a)}). \\ & (((p\ (ap\ V0P\ (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1l \in (ty.2Elist.2Elist \\ & A.27a). ((p\ (ap\ V0P\ V1l)) \Rightarrow (\forall V2x \in A.27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\ & c.2Elist.2ESNOC\ A.27a)\ V2x)\ V1l)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\ & A.27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (26)$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum. (\neg((ap\ c.2Enum.2ESUC\ V0n) = c.2Enum.2E0))) \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty.2Enum.2Enum}). (((p\ (ap\ V0P\ c.2Enum.2E0)) \wedge \\ & (\forall V1n \in ty.2Enum.2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c.2Enum.2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty.2Enum.2Enum. (p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (28)$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum. (\neg(p\ (ap\ (ap\ c.2Eprim_rec.2E.3C\ V0n)\ c.2Enum.2E0)))) \quad (29)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0l \in (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Erich_list_2EBUTLASTN \\
& \quad A_27a)\ c_2Enum_2E0)\ V0l) = V0l)) \wedge (\forall V1n \in ty_2Enum_2Enum. \\
& \quad (\forall V2x \in A_27b.(\forall V3l \in (ty_2Elist_2Elist\ A_27b). \\
& \quad (ap\ (ap\ (c_2Erich_list_2EBUTLASTN\ A_27b)\ (ap\ c_2Enum_2ESUC\ V1n)) \\
& \quad (ap\ (ap\ (c_2Elist_2ESNOC\ A_27b)\ V2x)\ V3l)) = (ap\ (ap\ (c_2Erich_list_2EBUTLASTN \\
& \quad \quad A_27b)\ V1n)\ V3l)))))) \\
& \hspace{15em} (30)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\
& \quad \forall V1l \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& V0n)\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1l))) \Rightarrow ((ap\ (c_2Elist_2ELENGTH \\
& \quad A_27a)\ (ap\ (ap\ (c_2Erich_list_2EBUTLASTN\ A_27a)\ V0n)\ V1l)) = (\\
& \quad ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1l)) \\
& \quad \quad V0n))))))
\end{aligned}$$