thm_2Erich_list_2ELENGTH_FOLDR (TMTyxjiRavLJNMhu5P3SpreKqv83uMvu7HS)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Enum\_2EREP\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2ESUC\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (omega^{omega}) \tag{3}$$

Let $c\_2Enum\_2EABS\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ ($

Let $c\_2Enum\_2EZERO\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \tag{5}$$

Definition 5 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{6}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{7}$$

Let $c\_2Elist\_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDR$$
$$A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27b})^{A\_27a})}) \tag{8}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{9}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist$$
$$A\_27a) \tag{10}$$

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.
$$True \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27b.((ap\ (\lambda V2x \in A\_27b. \tag{12}$$
$$V0t1)\ V1t2) = V0t1)))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in$$
$$A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow$$
$$True)) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)$$
$$(c\_2Elist\_2ENIL\ A\_27a)) = \mathsf{c\_2Enum\_2E0}) \land (\forall V0h \in A\_27a.($$
$$\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH$$
$$A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$(ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t)))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$(\forall V0f \in ((A\_27b^{A\_27b})^{A\_27a}).(\forall V1e \in A\_27b.((ap\ ($$
$$ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V0f)\ V1e)\ (c\_2Elist\_2ENIL$$
$$A\_27a)) = V1e))) \wedge (\forall V2f \in ((A\_27b^{A\_27b})^{A\_27a}).(\forall V3e \in$$
$$A\_27b.(\forall V4x \in A\_27a.(\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a).$$
$$((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c\_2Elist\_2ECONS$$
$$A\_27a)\ V4x)\ V5l)) = (ap\ (ap\ V2f\ V4x)\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR$$
$$A\_27a\ A\_27b)\ V2f)\ V3e)\ V5l))))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).$$
$$(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist$$
$$A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ ($$
$$c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist$$
$$A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{17}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist$$
$$A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR$$
$$A\_27a\ ty\_2Enum\_2Enum)\ (\lambda V1x \in A\_27a.(\lambda V2l\_27 \in ty\_2Enum\_2Enum.$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V2l\_27)))) \ \mathsf{c\_2Enum\_2E0})\ V0l)))$$