

thm_2Erich_list_2ELENGTH_NOT_NULL
(TMVJbt4PWG4wpRHA3fLD3xCbFLdVjNRaFmz)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{2}$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \tag{3}$$

Let $c_2Elist_2ENULL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENULL\ A_27a \in (2^{(ty_2Elist_2Elist\ A_27a)}) \tag{4}$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \tag{5}$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \tag{6}$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F_5C$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (8)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (9)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Definition 9 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 12 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (14)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow & (((ap (c.2Elist.2ELENGTH A.27a) \\ & (c.2Elist.2ENIL A.27a)) = c.2Enum.2E0) \wedge (\forall V0h \in A.27a. (\\ & \forall V1t \in (ty.2Elist.2Elist A.27a). ((ap (c.2Elist.2ELENGTH \\ A.27a) (ap (ap (c.2Elist.2ECONS A.27a) V0h) V1t)) = (ap c.2Enum.2ESUC \\ & (ap (c.2Elist.2ELENGTH A.27a) V1t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow & ((p (ap (c.2Elist.2ENULL A.27a) \\ & (c.2Elist.2ENIL A.27a))) \wedge (\forall V0h \in A.27a. (\forall V1t \in (\\ & ty.2Elist.2Elist A.27a). (\neg(p (ap (c.2Elist.2ENULL A.27a) (ap \\ & (ap (c.2Elist.2ECONS A.27a) V0h) V1t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow & (\forall V0P \in (2^{(ty.2Elist.2Elist A.27a)}). \\ & (((p (ap V0P (c.2Elist.2ENIL A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\ & A.27a). ((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A.27a. (p (ap V0P (ap (ap (\\ & c.2Elist.2ECONS A.27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\ & A.27a). (p (ap V0P V3l)))))) \end{aligned} \quad (19)$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum.(\neg(p (ap (ap c.2Eprim_rec.2E.3C V0n) c.2Enum.2E0)))) \quad (20)$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum.(p (ap (ap c.2Eprim_rec.2E.3C c.2Enum.2E0) (ap c.2Enum.2ESUC V0n)))) \quad (21)$$

Theorem 1

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0l \in (ty.2Elist.2Elist A.27a). ((p (ap (ap c.2Eprim_rec.2E.3C c.2Enum.2E0) (ap (c.2Elist.2ELENGTH A.27a) V0l))) \Leftrightarrow (\neg(p (ap (c.2Elist.2ENULL A.27a) V0l))))))$$