

# thm\_2Erich\_list\_2ELENGTH\_REPLICATE

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{2}$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{3}$$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{4}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{5}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (omega^{omega}) \tag{6}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{7}$$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 5** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num ($

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (8)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (9)$$

**Definition 6** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Erich\_list\_2EREPLICATE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Erich\_list\_2EREPLICATE A\_27a \in ((ty\_2Elist\_2Elist A\_27a)^{A\_27a})^{ty\_2Enum\_2Enum} \quad (10)$$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (12)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (((ap (c\_2Elist\_2ELENGTH A\_27a) (c\_2Elist\_2ENIL A\_27a)) = c\_2Enum\_2E0) \wedge (\forall V0h \in A\_27a. (\forall V1t \in (ty\_2Elist\_2Elist A\_27a). ((ap (c\_2Elist\_2ELENGTH A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V0h) V1t)) = (ap c\_2Enum\_2ESUC (ap (c\_2Elist\_2ELENGTH A\_27a) V1t))))))) \quad (14)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}). (((p (ap V0P c\_2Enum\_2E0)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum. (p (ap V0P V2n)))))) \quad (15)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & ((\forall V0x \in A\_27a. ((ap\ (ap\ (c\_2Erich\_list\_2EREPLICATE \\ & A\_27a)\ c\_2Enum\_2E0)\ V0x) = (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1n \in \\ & ty\_2Enum\_2Enum. (\forall V2x \in A\_27a. ((ap\ (ap\ (c\_2Erich\_list\_2EREPLICATE \\ & A\_27a)\ (ap\ c\_2Enum\_2ESUC\ V1n))\ V2x) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a) \\ & V2x)\ (ap\ (ap\ (c\_2Erich\_list\_2EREPLICATE\ A\_27a)\ V1n)\ V2x)))))) \\ & (16) \end{aligned}$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (\forall V0n \in ty\_2Enum\_2Enum. ( \\ \forall V1x \in A\_27a. & ((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (ap\ (c\_2Erich\_list\_2EREPLICATE \\ & A\_27a)\ V0n)\ V1x)) = V0n))) \end{aligned}$$