

thm_2Erich_list_2ELENGTH_SCANR
(TMGiEsv4JNgAqKy1xFdoP1NxxaoCDhtd6W8)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Definition 5 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{6}$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \tag{7}$$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EHD A_27a \in (A_27a^{(ty_2Elist_2Elist A_27a)}) \quad (8)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (9)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (10)$$

Let $c_2Erich_list_2ESCANR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Erich_list_2ESCANR A_27a A_27b \in (((ty_2Elist_2Elist A_27b)^{(ty_2Elist_2Elist A_27a)} A_27b)^{(A_27b^{A_27b} A_27a)}) \quad (11)$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (((ap (c_2Elist_2ELENGTH A_27a) (c_2Elist_2ENIL A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a.(\forall V1t \in (ty_2Elist_2Elist A_27a).((ap (c_2Elist_2ELENGTH A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V0h) V1t)) = (ap c_2Enum_2ESUC (ap (c_2Elist_2ELENGTH A_27a) V1t))))))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist A_27a)}).(((p (ap V0P (c_2Elist_2ENIL A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist A_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A_27a.(p (ap V0P (ap (ap (c_2Elist_2ECONS A_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist A_27a).(p (ap V0P V3l)))))) \quad (16)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad (\forall V0f \in ((A_27b^{A_27b})^{A_27a}). (\forall V1e \in A_27b. ((ap\ (\\
& \quad ap\ (ap\ (c_2Erich_list_2ESCANR\ A_27a\ A_27b)\ V0f)\ V1e)\ (c_2Elist_2ENIL \\
& \quad A_27a)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ V1e)\ (c_2Elist_2ENIL \\
& \quad A_27b)))))) \wedge (\forall V2f \in ((A_27b^{A_27b})^{A_27a}). (\forall V3e \in \\
& \quad A_27b. (\forall V4x \in A_27a. (\forall V5l \in (ty_2Elist_2Elist\ A_27a). \\
& \quad ((ap\ (ap\ (ap\ (c_2Erich_list_2ESCANR\ A_27a\ A_27b)\ V2f)\ V3e)\ (ap \\
& \quad (ap\ (c_2Elist_2ECONS\ A_27a)\ V4x)\ V5l)) = (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27b)\ (ap\ (ap\ V2f\ V4x)\ (ap\ (c_2Elist_2EHD\ A_27b)\ (ap\ (ap\ (ap\ (c_2Erich_list_2ESCANR \\
& \quad A_27a\ A_27b)\ V2f)\ V3e)\ V5l))))))\ (ap\ (ap\ (ap\ (c_2Erich_list_2ESCANR \\
& \quad A_27a\ A_27b)\ V2f)\ V3e)\ V5l))))))))) \\
& \hspace{15em} (17)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((A_27b^{A_27b})^{A_27a}). (\forall V1e \in A_27b. (\forall V2l \in \\
& \quad (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2ELENGTH\ A_27b)\ (ap \\
& \quad (ap\ (ap\ (c_2Erich_list_2ESCANR\ A_27a\ A_27b)\ V0f)\ V1e)\ V2l)) = (\\
& \quad ap\ c_2Enum_2ESUC\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V2l))))))
\end{aligned}$$