

thm_2Erich_list_2ELENGTH_SEG
(TMH9B1dbmioZdiUaE8NbK7xV8S6WsYxa2t3)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{2}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1t \in 2.V1t)) (\lambda V2t \in 2.V2t))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \tag{3}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{5}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 9 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge P$
of type $\iota \Rightarrow \iota$).

Definition 10 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P (ap (c_2Emin_2E40$

Definition 11 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21\ 2) (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (6)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (7)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (8)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (9)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Erich_list_2ESEG : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Erich_list_2ESEG\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (11)$$

Assume the following.

$$((\forall V0n \in ty_2Enum_2Enum.((ap (ap\ c_2Earithmetic_2E2B\ c_2Enum_2E0)\ V0n) = V0n)) \wedge (\forall V1m \in ty_2Enum_2Enum.(\forall V2n \in ty_2Enum_2Enum.((ap (ap\ c_2Earithmetic_2E2B\ (ap\ c_2Enum_2ESUC\ V1m))\ V2n) = (ap\ c_2Enum_2ESUC\ (ap (ap\ c_2Earithmetic_2E2B\ V1m)\ V2n)))))) \quad (12)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m)) \quad (13)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC V1n)))))) \quad (14)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0n)) (ap c_2Enum_2ESUC V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0n) V1m)))))) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (((ap\ (c.2Elist.2ELENGTH\ A.27a) \\ & (c.2Elist.2ENIL\ A.27a)) = c.2Enum.2E0) \wedge (\forall V0h \in A.27a. (\\ & \forall V1t \in (ty.2Elist.2Elist\ A.27a). ((ap\ (c.2Elist.2ELENGTH \\ A.27a)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a)\ V0h)\ V1t)) = (ap\ c.2Enum.2ESUC \\ & (ap\ (c.2Elist.2ELENGTH\ A.27a)\ V1t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0P \in (2^{(ty.2Elist.2Elist\ A.27a)}). \\ & (((p\ (ap\ V0P\ (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\ & A.27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\ c.2Elist.2ECONS\ A.27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\ & A.27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum. (\neg((ap\ c.2Enum.2ESUC\ V0n) = c.2Enum.2E0))) \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty.2Enum.2Enum}). (((p\ (ap\ V0P\ c.2Enum.2E0)) \wedge \\ & (\forall V1n \in ty.2Enum.2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c.2Enum.2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty.2Enum.2Enum. (p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (27)$$

Assume the following.

$$(\forall V0m \in ty.2Enum.2Enum. (\forall V1n \in ty.2Enum.2Enum. ((ap\ c.2Enum.2ESUC\ V0m) = (ap\ c.2Enum.2ESUC\ V1n)) \Leftrightarrow (V0m = V1n))) \quad (28)$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum. (\neg(p\ (ap\ (ap\ c.2Eprim_rec.2E.3C\ V0n)\ c.2Enum.2E0)))) \quad (29)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0k \in ty_2Enum_2Enum. \\
& (\forall V1l \in (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (ap\ (c_2Erich_list_2ESEG \\
& A_27a)\ c_2Enum_2E0)\ V0k)\ V1l) = (c_2Elist_2ENIL\ A_27a)))) \wedge ((\forall V2m \in \\
& ty_2Enum_2Enum.(\forall V3x \in A_27a.(\forall V4l \in (ty_2Elist_2Elist \\
& A_27a).((ap\ (ap\ (ap\ (c_2Erich_list_2ESEG\ A_27a)\ (ap\ c_2Enum_2ESUC \\
& V2m))\ c_2Enum_2E0)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3x)\ V4l)) = \\
& (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3x)\ (ap\ (ap\ (ap\ (c_2Erich_list_2ESEG \\
& A_27a)\ V2m)\ c_2Enum_2E0)\ V4l)))))) \wedge (\forall V5m \in ty_2Enum_2Enum. \\
& (\forall V6k \in ty_2Enum_2Enum.(\forall V7x \in A_27a.(\forall V8l \in \\
& (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (ap\ (c_2Erich_list_2ESEG \\
& A_27a)\ (ap\ c_2Enum_2ESUC\ V5m))\ (ap\ c_2Enum_2ESUC\ V6k))\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ A_27a)\ V7x)\ V8l)) = (ap\ (ap\ (ap\ (c_2Erich_list_2ESEG \\
& A_27a)\ (ap\ c_2Enum_2ESUC\ V5m))\ V6k)\ V8l)))))))))
\end{aligned} \tag{30}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\
& \forall V1k \in ty_2Enum_2Enum.(\forall V2l \in (ty_2Elist_2Elist \\
& A_27a).((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1k))\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V2l))) \Rightarrow ((ap\ (c_2Elist_2ELENGTH \\
& A_27a)\ (ap\ (ap\ (ap\ (c_2Erich_list_2ESEG\ A_27a)\ V0n)\ V1k)\ V2l)) = \\
& V0n))))))
\end{aligned}$$