

thm_2Erich__list_2ELENGTH__UNZIP__SND
(TMc8ETSs33A4VuF1QNTUT4DKeWMYFpVZgrm)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V0t \in 2.V0t)) (\lambda V1t \in 2.V1t))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 7 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (c_2Enum_2ESUC_REP m))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (6)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (7)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (8)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (9)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (10)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (11)$$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (12)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ x\ y))$

Let $c_2Elist_2EUNZIP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EUNZIP\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ (ty_2Elist_2Elist\ A_27a)\ (ty_2Elist_2Elist\ A_27b))^{(ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b))}) \quad (13)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (14)$$

Definition 11 We define $c_2Erich_list_2EUNZIP_SND$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0l \in (ty_2Elist_2Elist$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \tag{18}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2ELENGTH\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a. (\forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2ELENGTH\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ c_2Enum_2ESUC\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1t)))))) \tag{19}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist\ A_27a). (p\ (ap\ V0P\ V3l)))))) \tag{20}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& ((ap\ (c.2Elist.2EUNZIP\ A.27a\ A.27b)\ (c.2Elist.2ENIL\ (ty.2Epair.2Eprod \\
& \quad A.27a\ A.27b))) = (ap\ (ap\ (c.2Epair.2E.2C\ (ty.2Elist.2Elist\ A.27a) \\
& \quad (ty.2Elist.2Elist\ A.27b))\ (c.2Elist.2ENIL\ A.27a))\ (c.2Elist.2ENIL \\
& \quad A.27b))) \wedge (\forall V0x \in (ty.2Epair.2Eprod\ A.27a\ A.27b). (\forall V1l \in \\
& (ty.2Elist.2Elist\ (ty.2Epair.2Eprod\ A.27a\ A.27b)). ((ap\ (c.2Elist.2EUNZIP \\
& \quad A.27a\ A.27b)\ (ap\ (ap\ (c.2Elist.2ECONS\ (ty.2Epair.2Eprod\ A.27a \\
& \quad A.27b))\ V0x)\ V1l)) = (ap\ (ap\ (c.2Epair.2E.2C\ (ty.2Elist.2Elist \\
& \quad A.27a)\ (ty.2Elist.2Elist\ A.27b))\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a) \\
& (ap\ (c.2Epair.2EFST\ A.27a\ A.27b)\ V0x))\ (ap\ (c.2Epair.2EFST\ (ty.2Elist.2Elist \\
& \quad A.27a)\ (ty.2Elist.2Elist\ A.27b))\ (ap\ (c.2Elist.2EUNZIP\ A.27a \\
& \quad A.27b)\ V1l))))))\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27b)\ (ap\ (c.2Epair.2ESND \\
& \quad A.27a\ A.27b)\ V0x))\ (ap\ (c.2Epair.2ESND\ (ty.2Elist.2Elist\ A.27a) \\
& \quad (ty.2Elist.2Elist\ A.27b))\ (ap\ (c.2Elist.2EUNZIP\ A.27a\ A.27b) \\
& \quad V1l))))))))) \\
& \hspace{15em} (21)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0x \in A.27a. (\forall V1y \in A.27b. ((ap\ (c.2Epair.2ESND\ A.27a \\
& \quad A.27b)\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V1y))) \\
& \hspace{15em} (22)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0l \in (ty.2Elist.2Elist\ (ty.2Epair.2Eprod\ A.27a\ A.27b)). \\
& ((ap\ (c.2Elist.2ELENGTH\ A.27b)\ (ap\ (c.2Erich_list.2EUNZIP_SND \\
& \quad A.27b\ A.27a)\ V0l)) = (ap\ (c.2Elist.2ELENGTH\ (ty.2Epair.2Eprod \\
& \quad A.27a\ A.27b))\ V0l)))
\end{aligned}$$