

thm_2Erich__list_2EMAP__FOLDR (TMHvjMEpxBQufS9vz5qZbMfFz3uEbQPLfW6)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EMAP A_27a A_27b \in (((ty_2Elist_2Elist A_27b)^{(ty_2Elist_2Elist A_27a)})^{(A_27b^{A_27a})}) \quad (2)$$

Let $c_2Elist_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EFOLDR A_27a A_27b \in (((A_27b^{(ty_2Elist_2Elist A_27a)})^{A_27b})^{((A_27b^{A_27b})^{A_27a})}) \quad (3)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (5)$$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$
Assume the following.

$$True \tag{6}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{7}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{8}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & (\forall V0f \in (A_27b^{A_27a}).((ap (ap (c_2Elist_2EMAP A_27a A_27b) \\ & V0f) (c_2Elist_2ENIL A_27a)) = (c_2Elist_2ENIL A_27b))) \wedge (\forall V1f \in \\ & (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\ & A_27a).((ap (ap (c_2Elist_2EMAP A_27a A_27b) V1f) (ap (ap (c_2Elist_2ECONS \\ & A_27a) V2h) V3t)) = (ap (ap (c_2Elist_2ECONS A_27b) (ap V1f V2h)) \\ & (ap (ap (c_2Elist_2EMAP A_27a A_27b) V1f) V3t))))))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & (\forall V0f \in ((A_27b^{A_27b})^{A_27a}).(\forall V1e \in A_27b.((ap (\\ & ap (ap (c_2Elist_2EFOLDR A_27a A_27b) V0f) V1e) (c_2Elist_2ENIL \\ & A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b^{A_27b})^{A_27a}).(\forall V3e \in \\ & A_27b.(\forall V4x \in A_27a.(\forall V5l \in (ty_2Elist_2Elist A_27a). \\ & ((ap (ap (ap (c_2Elist_2EFOLDR A_27a A_27b) V2f) V3e) (ap (ap (c_2Elist_2ECONS \\ & A_27a) V4x) V5l)) = (ap (ap V2f V4x) (ap (ap (c_2Elist_2EFOLDR \\ & A_27a A_27b) V2f) V3e) V5l))))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist A_27a)}). \\ & (((p (ap V0P (c_2Elist_2ENIL A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A_27a.(p (ap V0P (ap (ap (\\ & c_2Elist_2ECONS A_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a).(p (ap V0P V3l)))))) \end{aligned} \tag{11}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & (\forall V0f \in (A_27b^{A_27a}).(\forall V1l \in (ty_2Elist_2Elist A_27a). \\ & ((ap (ap (c_2Elist_2EMAP A_27a A_27b) V0f) V1l) = (ap (ap (ap (c_2Elist_2EFOLDR \\ & A_27a (ty_2Elist_2Elist A_27b)) (\lambda V2x \in A_27a.(\lambda V3l_27 \in \\ & (ty_2Elist_2Elist A_27b).(ap (ap (c_2Elist_2ECONS A_27b) (ap \\ & V0f V2x)) V3l_27)))) (c_2Elist_2ENIL A_27b)) V1l)))) \end{aligned}$$