

# thm\_2Erich\_list\_2EMEM\_BUTLASTN (TMZA- xVHuvQ4NLHdEnBgYE1BJ1u1voVSWbQj)

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Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let  $c\_2Elist\_2ELIST\_TO\_SET : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2ELIST\_TO\_SET\ A.27a \in \left( (2^{A.27a})^{(ty\_2Elist\_2Elist\ A.27a)} \right) \quad (2)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A.27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A.27a}).(ap\ V1f\ V0x)))$

Let  $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2EREVERSE\ A.27a \in \left( (ty\_2Elist\_2Elist\ A.27a)^{(ty\_2Elist\_2Elist\ A.27a)} \right) \quad (3)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (4)$$

Let  $c\_2Elist\_2EDROP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2EDROP\ A.27a \in \left( (ty\_2Elist\_2Elist\ A.27a)^{(ty\_2Elist\_2Elist\ A.27a)} ty\_2Enum\_2Enum \right) \quad (5)$$

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A.27a}))))$

**Definition 5** We define  $c\_2Erich\_list\_2EBUTLASTN$  to be  $\lambda A.27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1xs \in$

Let  $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEXISTS\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \quad (6)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (7)$$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (10)$$

**Definition 10** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

**Definition 11** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A \wedge P\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 12** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a)\ P)))$

**Definition 13** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V1n$

**Definition 14** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

**Definition 15** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V1n$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1l \in \\ & (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x) \\ & (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ V1l)))) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS \\ & A\_27a)\ (ap\ (c\_2Emin\_2E\_3D\ A\_27a)\ V0x))\ V1l)))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum.( \\
& \quad \forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V0m)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))) \Rightarrow (\forall V2P \in (2^{A\_27a}). \\
& \quad ((p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V2P)\ (ap\ (ap\ (c\_2Erich\_list\_2EBUTLASTN \\
& \quad A\_27a)\ V0m)\ V1l))) \Rightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V2P)\ V1l))))))
\end{aligned}
\tag{12}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum.( \\
& \quad \forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V0m)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))) \Rightarrow (\forall V2x \in A\_27a. \\
& \quad ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET \\
& \quad A\_27a)\ (ap\ (ap\ (c\_2Erich\_list\_2EBUTLASTN\ A\_27a)\ V0m)\ V1l)))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET \\
& \quad A\_27a)\ V1l))))))
\end{aligned}$$