

thm\_2Erich\_\_list\_2EMEM\_\_DROP\_\_IMP  
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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 6** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 8** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \tag{3}$$

**Definition 9** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2Epair\_2EABS\_prod$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (4)$$

Let  $c\_2Elist\_2ELIST\_TO\_SET : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2ELIST\_TO\_SET\ A.27a \in ((2^{A.27a})^{(ty\_2Elist\_2Elist\ A.27a)}) \quad (5)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (6)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (7)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (8)$$

**Definition 10** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 11** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (10)$$

**Definition 12** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 13** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n))\ c\_2Enum\_2E0)$

**Definition 14** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 15** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E.21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 16** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 17** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Elist\_2ECONS A.27a \in (((ty\_2Elist\_2Elist A.27a)^{(ty\_2Elist\_2Elist A.27a)} A.27a) \quad (13)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Elist\_2ENIL A.27a \in (ty\_2Elist\_2Elist A.27a) \quad (14)$$

Let  $c\_2Elist\_2EDROP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Elist\_2EDROP A.27a \in (((ty\_2Elist\_2Elist A.27a)^{(ty\_2Elist\_2Elist A.27a)} ty\_2Enum\_2Enum) \quad (15)$$

**Definition 18** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c\_2Ebool\_2E\_EF).$

**Definition 19** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_EF).$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A.27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ p V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0t1 \in A.27a.(\forall V1t2 \in \\ A.27a.(((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2ET) V0t1) \\ V1t2) = V0t1) \wedge ((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2EF \\ V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee \\ (p V0A)))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\ 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow \\ (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0h \in A\_27b. (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27b). ( \\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = \\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)) \wedge ((ap\ (c\_2Elist\_2ELIST\_TO\_SET \\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c\_2Epred\_set\_2EINSERT \\ A\_27b)\ V0h)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27b)\ V1t)))))) \\ (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). \\ (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\ A\_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a. (p\ (ap\ V0P\ (ap\ (ap\ ( \\ c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ A\_27a). (p\ (ap\ V0P\ V3l)))))) \\ (32) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a. ((p\ (ap\ (ap \\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a) \\ (c\_2Elist\_2ENIL\ A\_27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A\_27a. (\forall V2h \in \\ A\_27a. (\forall V3t \in (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ A\_27a)\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\ A\_27a)\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ V3t)))))))) \\ (33) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0n \in ty\_2Enum\_2Enum. \\ ((ap\ (ap\ (c\_2Elist\_2EDROP\ A\_27a)\ V0n)\ (c\_2Elist\_2ENIL\ A\_27a)) = \\ (c\_2Elist\_2ENIL\ A\_27a)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2x \in \\ A\_27a. (\forall V3xs \in (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (ap\ (c\_2Elist\_2EDROP \\ A\_27a)\ V1n)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x)\ V3xs)) = (ap\ (ap \\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Elist\_2Elist\ A\_27a)\ (ap\ (ap\ (c\_2Emin\_2E\_3D \\ ty\_2Enum\_2Enum)\ V1n)\ c\_2Enum\_2E0))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a) \\ V2x)\ V3xs))\ (ap\ (ap\ (c\_2Elist\_2EDROP\ A\_27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D \\ V1n)\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\ c\_2Earithmetic\_2EZERO))))\ V3xs)))))) \\ (34) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\neg (p\ (ap\ (ap \\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))) \\ (35) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.((p \vee 0A) \Rightarrow ((\neg(p \vee 0A)) \Rightarrow \text{False}))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \vee 0A) \vee (p \vee 1B))) \Rightarrow \text{False}) \Leftrightarrow ((p \vee 0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p \vee 1B)) \Rightarrow \text{False})))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \vee 0A)) \vee (p \vee 1B))) \Rightarrow \text{False}) \Leftrightarrow ((p \vee 0A) \Rightarrow ((\neg(p \vee 1B)) \Rightarrow \text{False})))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \vee 0A)) \Rightarrow \text{False}) \Rightarrow (((p \vee 0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \Leftrightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee ((p \vee 1q) \vee (p \vee 2r))) \wedge (((p \vee 0p) \vee ((\neg(p \vee 2r)) \vee (\neg(p \vee 1q)))) \wedge (((p \vee 1q) \vee ((\neg(p \vee 2r)) \vee (\neg(p \vee 0p)))) \wedge ((p \vee 2r) \vee ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p)))))))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \wedge (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee ((\neg(p \vee 1q)) \vee (\neg(p \vee 2r)))) \wedge (((p \vee 1q) \vee (\neg(p \vee 0p))) \wedge ((p \vee 2r) \vee (\neg(p \vee 0p)))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \vee (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (\neg(p \vee 1q))) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge ((p \vee 1q) \vee ((p \vee 2r) \vee (\neg(p \vee 0p)))))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \Rightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge ((\neg(p \vee 1q)) \vee ((p \vee 2r) \vee (\neg(p \vee 0p)))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \vee 0p) \Leftrightarrow (\neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p)))))) \quad (45)$$

**Theorem 1**

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ A_{27a}). (\forall V1m \in ty\_2Enum\_2Enum. (\forall V2x \in A_{27a}. ((p \\ (ap (ap (c\_2Ebool\_2EIN A_{27a}) V2x) (ap (c\_2Elist\_2ELIST\_TO\_SET \\ A_{27a}) (ap (ap (c\_2Elist\_2EDROP A_{27a}) V1m) V0l)))) \Rightarrow (p (ap (ap ( \\ c\_2Ebool\_2EIN A_{27a}) V2x) (ap (c\_2Elist\_2ELIST\_TO\_SET A_{27a}) \\ V0l))))))))))$$