thm_2Erich_list_2EMEM__FOLDL_MAP (TMKJUPysR-jty1EtCHmb2KJLTeoASnWsG8sQ)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \qquad (1)$$

Let $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EMAP$$
$$A\_27a\ A\_27b \in (((ty\_2Elist\_2Elist\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(A\_27b^{A\_27a})}) \qquad (2)$$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t)).$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDL$$
$$A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27a})^{A\_27b})}) \qquad (3)$$

Let $c\_2Elist\_2ELIST\_\_TO\_\_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a \in$$
$$((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \qquad (4)$$

Definition 7 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.$$
$$nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27a^{A\_27b})^{A\_27a}).(\forall V1e \in$$
$$A\_27a.(\forall V2g \in (A\_27b^{A\_27c}).(\forall V3l \in (ty\_2Elist\_2Elist$$
$$A\_27c).((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27b\ A\_27a)\ V0f)\ V1e)\ (ap$$
$$(ap\ (c\_2Elist\_2EMAP\ A\_27c\ A\_27b)\ V2g)\ V3l)) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL$$
$$A\_27c\ A\_27a)\ (\lambda V4x \in A\_27a.(\lambda V5y \in A\_27c.(ap\ (ap\ V0f\ V4x)\ ($$
$$ap\ V2g\ V5y))))) \ V1e)\ V3l)))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0y \in A\_27a.(\forall V1l \in$$
$$(ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0y)$$
$$(ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ V1l))) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL$$
$$A\_27a\ 2)\ (\lambda V2l\_27 \in 2.(\lambda V3x \in A\_27a.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_5C\_2F}$$
$$V2l\_27)\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V0y)\ V3x))))) \ \mathsf{c\_2Ebool\_2EF})$$
$$V1l))))) \tag{9}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1l \in$$
$$(ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)$$
$$(ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ V1l))) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL$$
$$2\ 2)\ \mathsf{c\_2Ebool\_2E\_5C\_2F})\ \mathsf{c\_2Ebool\_2EF})\ (ap\ (ap\ (c\_2Elist\_2EMAP$$
$$A\_27a\ 2)\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V0x))\ V1l))))))$$