

thm_2Erich__list_2EMONOID__APPEND__NIL
(TMMKwVwP1574j8ytnxu6xgwcBJefN4yuKeW)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Ecombin_2ELEFT_ID$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27b^{A_27b})^{A_27a}).\lambda$

Definition 6 We define $c_2Ecombin_2ERIGHT_ID$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27b})^{A_27a}).\lambda$

Definition 7 We define $c_2Ecombin_2EASSOC$ to be $\lambda A_27a : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27a}).(ap (c_2Ebool$

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 10 We define $c_2Ecombin_2EMONOID$ to be $\lambda A_27a : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27a}).\lambda V1e \in A$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (2)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (3)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)(ty_2Elist_2Elist\ A_27a))(ty_2Elist_2Elist\ A_27a)) \quad (4)$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (6)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (7) \end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) \\ & V0l) = V0l)) \wedge (\forall V1l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V2l2 \in \\ & (ty_2Elist_2Elist\ A_27a).(\forall V3h \in A_27a.((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a) \\ & (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a) \\ & V1l1)\ V2l2))))))) \quad (9) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V1l2 \in (ty_2Elist_2Elist\ A_27a).(\forall V2l3 \in \\ & (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a) \\ & V0l1)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V1l2)\ V2l3)) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a) \\ & (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ V1l2))\ V2l3)))))) \quad (10) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0l \in (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l)\ (c_2Elist_2ENIL\ A_27a)) = V0l)) \wedge (\forall V1l \in (ty_2Elist_2Elist\ A_27b).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27b)\ (c_2Elist_2ENIL\ A_27b)) \\ & V1l) = V1l))) \quad (11) \end{aligned}$$

Theorem 1

$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (p\ (ap\ (ap\ (c_2Ecombin_2EMONOID\ (ty_2Elist_2Elist\ A_{27a}))\ (c_2Elist_2EAPPEND\ A_{27a}))\ (c_2Elist_2ENIL\ A_{27a})))$